Annual Review of Statistics and Its Application

Statistical Modeling of the Ocean Environment

Erik Vanem,[1] Sheng Dong,[2] Guillaume de Hauteclocque,[3] Thomas Berge Johannessen,[1] Tingyao Zhu,[4] Jasna Prpic-Orsic,[5] Sanne van Essen,[6,7] Kevin Ewans,[8,9] Ed Mackay,[10] and Philip Jonathan[11]

[1]DNV, Høvik, Norway; email: erik.vanem@dnv.com

[2]Department of Ocean Engineering, Ocean University of China, Qingdao, China

[3]Bureau Veritas Marine and Offshore, Courbevoie, France

[4]Nippon Kaiji Kyokai, Tokyo, Japan

[5]Faculty of Engineering, University of Rijeka, Rijeka, Croatia

[6]Maritime Research Institute Netherlands (MARIN), Wageningen, The Netherlands

[7]Faculty of Mechanical Engineering, Delft University of Technology, Delft, The Netherlands

[8]MetOcean Research Ltd., New Plymouth, New Zealand

[9]Department of Infrastructure Engineering, University of Melbourne, Melbourne, Victoria, Australia

[10]Department of Engineering, University of Exeter, Penryn, United Kingdom

[11]School of Mathematical Sciences, Lancaster University, Lancaster, United Kingdom

Annu. Rev. Stat. Appl. 2026. 13:1–28

The *Annual Review of Statistics and Its Application* is online at statistics.annualreviews.org

https://doi.org/10.1146/annurev-statistics-042424-115755

Keywords

ocean environment, wave statistics, risk and reliability, probabilistic models, extreme value analysis, multivariate statistics, spatial statistics, temporal statistics, nonstationarity

Abstract

Statistical modeling of the ocean environment is important for many practical applications in science and engineering. Probabilistic descriptions of the ocean environment are important input for structural design and risk assessment of marine structures, including ships, offshore and coastal structures, and aquaculture installations. They are also essential for the safe operation of ships and other structures at sea. Additionally, they are critical for planning and decision-making in the exploitation of marine renewable energy sources such as waves, tides, and offshore wind. This article presents a review of recent developments with regard to statistical modeling of the ocean

environment, with a particular focus on ocean waves. Such developments are driven by an increasing volume of available data, increasing computational capabilities, and demand from the industry for more accurate and uncertainty-aware descriptions of relevant oceanic variables. Hence, statistical modeling of the ocean environment remains an active area of research, with significant developments in various directions. These are reviewed in this article.

1. INTRODUCTION AND BACKGROUND

Probabilistic descriptions of the ocean environment, informed by statistical modeling of data, are important for the design and structural reliability assessment of ships and other marine structures. They provide the necessary input to identify what conditions the structure is expected to experience and hence what environmental loads it should be designed to withstand. At a high level, this is illustrated by the reliability equation (Equation 1), where S are loads, R is the structural resistance of the structure, P_f is its probability of failure, and g_l is called the limit state function:

$$P_f = \mathrm{P}(g_l \leq 0) = \mathrm{P}\left((R - S) \leq 0\right). \tag{1}$$

More accurate statistical modeling of the loads can allow for more optimized design, where unnecessary conservatism can be avoided by reducing safety factors and associated building costs (Bitner-Gregersen et al. 2022). Moreover, probabilistic descriptions of the ocean environment are important for assessing the potential for marine renewable energy from offshore wind and waves and hence for the design and operation of marine renewable energy installations. In coastal areas, probabilistic models with a special focus on shallow-water conditions are needed for coastal engineering applications. This article reviews recent developments in the theory and application of statistical modeling of relevant environmental variables that describe ocean conditions. The focus is mostly on statistical modeling applied to such data, but important theoretical and methodological developments are also included if deemed relevant.

Recent developments in statistical modeling of the ocean environment for engineering applications have resulted in more advanced probabilistic models, and these developments are driven by several factors. The increasing access to relevant environmental data—from observations, numerical models, hindcasts, and laboratory experiments—along with improvements in resolution, accuracy, and spatial and temporal coverage, combined with enhanced computational capabilities, enables the development of more sophisticated statistical models. These include nonstationary models that can take covariate effects into account and multivariate models in increasing dimensions for the joint behavior of several metocean variables. This enables a more precise probabilistic description of the ocean environment. This description is an important basis for decision-making and can lead to more optimized design and operation of marine structures. Another trend is toward more fully probabilistic approaches, where both aleatory and epistemic uncertainties are propagated through several links in the modeling chain. Bayesian approaches to statistical modeling are useful tools in this regard and have gained increasing interest in recent years.

The article addresses different aspects of statistical modeling and discusses the following topics: long- and short-term statistics, extreme value analysis, multivariate analysis, spatio-temporal analysis, and nonstationary analysis. While this list of topics may be somewhat arbitrary, as some papers contribute to multiple aspects, it is still a useful framework for organization. The review is limited to recent developments, and we refer readers to Vanem et al. (2022) and Babanin et al. (2022) for previous reviews.

2. LONG- AND SHORT-TERM STATISTICS

Environmental processes are often assumed piecewise stationary for modeling purposes. A long-term probabilistic description of environmental variables often combines a model for the long-term variation of environmental conditions (e.g., integrated variables describing sea states or wind conditions) with a conditional model for their short-term variability (e.g., individual wave heights and instantaneous wind speeds). This assumption is reflected by, e.g., Equation 2 (from Haver & Winterstein 2008), where $1 - F_{X_d}(x)$ is the exceedance probability of the ship response or load X during duration d, $\left(1 - F_{X_d|H_s,T_p}(x|h,t)\right)$ is its short-term distribution (given a sea state characterized by wave height H_s and wave period T_p), and $f_{H_s,T_p}(h,t)$ is the long-term distribution of these sea states:

$$1 - F_{X_d}(x) = \int_h \int_t \left(1 - F_{X_d|H_s,T_p}(x|h,t)\right) f_{H_s,T_p}(h,t)\,\mathrm{d}t\,\mathrm{d}h. \qquad 2.$$

This section presents a review of statistical modeling of both long-term environmental conditions and short-term environmental variables.

2.1. Statistics for Arbitrary Water Depths

A recent review of probabilistic models for environmental conditions, including wind and wave parameters, was presented by Ramezani et al. (2023). Qin (2022) presented challenges in the probabilistic modeling for long-term significant wave height H_s and also formulated a novel probabilistic framework for long-term H_s, which identifies homogeneous clusters of wave records, including a definition of the boundary and a probabilistic description of different clusters.

The statistical distribution of wave crest elevation is a key input for the design and operation of offshore structures. For example, in offshore engineering, the estimated crest height at a specified probability of exceedance for the defined duration or the design return period is used to define the deck elevation, maintain an effective air gap, and avoid wave-in-deck loading. The long- and short-term statistics of wave crests therefore remain an active area of research. van Essen et al. (2023a) and Scharnke et al. (2023) investigated how many seed variations of a short-term sea state are needed in tank tests to derive a reliable and repeatable probability distribution of the design wave and extreme design loads (such as strongly nonlinear green water loading, slamming, or air gap impacts). Based on their results, they provided guidelines for the convergence of most probable maximum (MPM) wave crest heights, MPM green water wave impact forces, and MPM wave-in-deck loads on a stationary deck box and a ferry. Moreover, Zve et al. (2023) explored the competing nonlinear processes that define the largest crest heights in unidirectional random seas. They explored how near-resonant interactions affect the crest heights arising in broad-banded, nonbreaking, unidirectional seas in a wide range of effective water depths. They also quantified the role of the bound-wave interactions. They calculated that $k_p d = 1.363$ (k_p being the wavenumber of the spectral peak frequency and d the water depth) indeed defines the boundary between energy focusing and defocusing for realistic JONSWAP sea states, irrespective of the spectral bandwidth and steepness. However, they also concluded that the bound-wave contributions increased the largest crest heights, while the near-resonant interactions reduced them for $k_p d < 1.363$. These results have important implications for describing crest height distributions and for the appropriateness of second-order wave models in engineering. Furthermore, Petrova et al. (2022) evaluated the probabilistic structure of experimental irregular unidirectional sea states that are subjected to the effects of nonlinear focusing. They also investigated the evolution of the statistical distributions in space and time, focusing on the crest maxima of waves exceeding a prescribed threshold, to improve modeling and prediction of extremely large waves. Note that the findings on wave crest distributions in these experimental studies may be subject to some differences with conditions at sea, as waves can only be generated on the sides of the basin.

Wave parameters such as the skewness and kurtosis are often used in the evaluation of nonlinear extreme waves such as freak/rogue waves. Gramstad & Lian (2024) therefore calculated the skewness and kurtosis for a large number of sea states, covering a wide range of covariates (steepness, water depth, directional spreading, and frequency bandwidth). Based on their results, they developed an efficient and convenient numerical method for calculation of the sea surface skewness and kurtosis for arbitrary wave spectra, to include these covariates into higher-order distributions for crest heights, wave heights, and surface elevation. For free surface elevation of a nonlinear irregular water wave field, Fuhrman et al. (2023) revisited the derivation of the probability density function (pdf) of Longuet-Higgins (1952), utilizing both moment and cumulant generating functions. They found that the second-order pdf can be represented exactly in terms of the Airy function through a change of variables coupled with complex analysis. The second-order pdf modified by them predicted increased probability of extreme positive surface elevations typical of, e.g., rogue waves with good accuracy for directionally spread irregular seas in both finite and infinite water depths.

A variety of short-term wave height distribution models are available (see, e.g., **Figure 1**); a recent study investigated the goodness of fit of various short-term crest distributions to data with varying sea-state conditions (Vanem et al. 2024b). Each of these models is successful in at least one water depth regime, but not always in intermediate and shallow water depths or in steep sea states due to the effects of wave breaking.

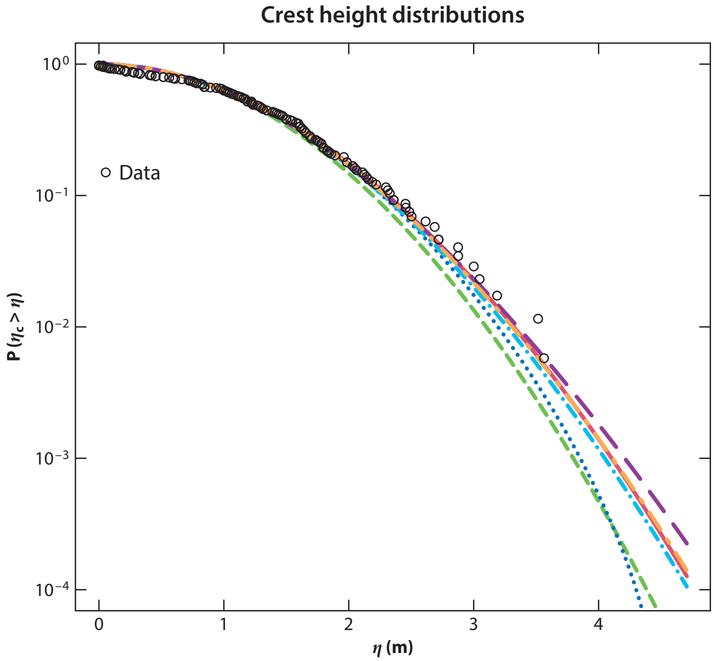

Figure 1

Comparing various state-of-the-art crest height distributions to data. Circles denote observations and lines represent different candidate models fitted to the data.

2.2. Shallow-Water Statistics

Ocean waves undergo significant modification as they propagate into shallow water, and the effects that this has on their statistics have been a long-standing subject of interest. These include wave heights' amplification due to shoaling and their attenuation through various dissipative effects, such as bottom friction and breaking. These factors influence short-term distributions as individual waves lose energy through highly nonlinear processes. They also impact long-term statistics, particularly by imposing limits on wave heights due to dissipation.

Wu et al. (2016) proposed a two-part Weibull–generalized Pareto (GP) model for wave height in shallow water, based on data from both laboratory and field measurements, incorporating the Miche (1944) upper limit for wave heights that accounts for the effect of breaking waves. This distribution, among others, was included by Karmpadakis et al. (2022) when evaluating the performance of a new wave height distribution model they proposed for finite water depths. Their new model considered nonlinearity, directional spreading, water depth, and spectral bandwidth effects. The model was also validated against a number of data sets from measurements made in intermediate- and shallow-water depth locations in the central and southern regions of the North Sea. They concluded that their proposed model performed well over a wide range of conditions and was a significant improvement over the other models it was compared with.

Nonlinear interactions above second order and wave breaking were also incorporated into a new crest height distribution proposed by Karmpadakis & Swan (2022). This formulation is provided in Equation 3, where $\eta^{(1)}$ is the linear Rayleigh distribution for crest heights, κ is a fitted function that describes higher-order crest height effects, μ is a measure for wave steepness considering nonlinear effects, and $(A\chi + B)$ parametrizes wave breaking (see Karmpadakis & Swan 2022 for values of μ, A, and B):

$$\zeta = \frac{\eta_M}{H_s} = \left(\chi + 2\mu\chi^2 + \kappa\mu\chi\right) \cdot (A\chi + B),$$

$$\text{where: } \chi = \frac{\eta^{(1)}}{H_s}, \qquad 3.$$

$$\kappa = \frac{1}{1 + k^3 \exp(-10k\mu)}, \text{ with: } k = 25.3.$$

The database of measurements used by Karmpadakis et al. (2022) was extended to include deep-water sites in the northern North Sea, and an extensive set of experimental measurements was used to validate the proposed new distribution. It was found that the new model performed better than other models it was compared with, particularly in the steepest, most severe sea states. The new formulation was validated for conditions with effective water depths ranging from shallow to deep water ($0.5 < k_p d < 3$) and sea-state steepnesses covering mild, severe, and extreme conditions. In an attempt to more explicitly account for the physics of breaking waves, Hossain et al. (2022) included a description of entrained air bubbles on a sloping beach in a proposed new distribution for wave heights. The model performed better than the Rayleigh and Mendez et al. (2004) distributions when validated with field data. Hossain et al. (2024) commented that the Hossain et al. (2022) distribution was limited by not including the effect of air bubbles on plunging breaking waves, and it was more effective for small rather than large wave heights. Accordingly, Hossain et al. (2024) developed a new wave height distribution model just seaward of the breaking zone for waves impacted by air bubble effects in plunging–breaking scenarios. The model was found to perform better than the Rayleigh model when assessed on experimental data, but a fuller evaluation of its overall performance must await comparison with other shallow-water models and with field data.

The impact of seabed irregularities on the statistics of waves propagating into shallow water has been the focus of several recent studies, with particular interest in the potential occurrence of rogue waves. Bonar et al. (2021) were unsuccessful in modeling the effect of abrupt changes in seabed bathymetry on the skewness and kurtosis of free surface elevation, using a Boussinesq-type formulation. Lawrence & Gramstad (2020) found that both the surface elevation and the horizontal velocity field have local maxima in skewness and kurtosis when modeling long-crested irregular waves propagating over a circular shoal and submerged bar using the higher-order spectral (HOS) model (Gouin et al. 2016). Using numerical data produced earlier by Zhang et al. (2022), Zhang et al. (2024) studied the effects of propagation over a steep shoal on the probability distributions of free surface elevation and wave height, as well as on statistical moments and maximum wave statistics of unidirectional irregular wave trains. They showed that the statistics in the far-field region are significantly influenced by the near-field wave–wave interaction and could not be predicted by the considered statistical models. In particular, they also found that a rogue wave in the sea state in the far-field region of a steep shoal was less likely than in a Gaussian sea state.

The long-term statistics of sea-state parameters, such as H_s and mean wave periods, are expected to be affected by shallow-water effects. The limitation in H_s may be inherently accounted for when fitting long-term data sets with an extremal distribution, such as the GP distribution, where a negative shape parameter imposes an upper bound. Nevertheless, Vanem & Fazeres-Ferradosa (2022) proposed a truncated, translated Weibull distribution for the long-time distribution of H_s in shallow water and demonstrated that the model fits shallow-water data quite well.

3. EXTREME VALUE ANALYSIS AND EXTREME WAVE STATISTICS

Extreme weather events reported over the past decade have attracted the attention of scientists and experts in the shipping and offshore industries, because they are closely related to safety at sea. We discuss advances in the estimation of extreme significant wave height, as well as in the estimation of the associated uncertainty and variability. In the design of maritime structures, it should be acknowledged that the distributions of wave extremes and wave-induced response extremes are not necessarily identical. This implies that the largest wave in a given duration is not always associated with the largest wave-induced response (e.g., ship roll angle or wave impact load on a deck structure). Methods to account for this are also discussed briefly below, as well as some software implementations.

An important step that often occurs in design is the estimation of an extreme long-term design value for significant wave height, based on recorded or hindcast data (e.g., Soares et al. 1996). This usually involves fitting a suitable probability distribution to the wave height data and extrapolating this distribution to a given return period to determine a suitable design wave height, e.g., the so-called 100- or 50-year wave. This is a characteristic large wave height that can be expected with a certain low probability during the lifetime of the structure. Some important statistical problems can be distinguished in the prediction of wave extremes. Firstly, the predicted extreme values for, say, 100 years must be based on data collected over a relatively short period, for example, 10 years. Secondly, the observed data must be extrapolated into their extreme region, typically lying well beyond the maxima of the available observations.

This fitting and extrapolation is often done using some form of the generalized extreme value (GEV) distribution of Equation 4, where μ, σ, and ξ are the location, scale, and shape parameters, respectively:

$$f(x) = \frac{1}{\sigma}\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi-1}\exp\left[-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi}\right]. \qquad 4.$$

It is valid for block-maximum or annual-maximum (AM) data. Depending on the values of these parameters, the GEV can be simplified to the Gumbel, Weibull, or Fréchet family of distributions. The GP distribution is a special case of the GEV, suitable for peak-over-threshold (POT) data. de Hauteclocque et al. (2023) proposed a new fitting technique for the probability distribution of H_s. The model is a combination of three different distributions, fitted separately to the core and extreme values of the data. For the core, the authors recommend a mixed distribution with two components, log-normal and Weibull, while the tail is fitted with the GP distribution. For the conditional distribution of the wave periods, the model is based on a generalized split normal distribution with the corresponding dependency functions.

Improved knowledge of the uncertainties in extreme value prediction methods has a direct impact on reducing the safety factor, and consequently the cost, of marine structures. The uncertainties of wave data and models are therefore receiving increasing attention as well. The article by Bitner-Gregersen et al. (2022), developed by the ISSC-ITTC (International Ship and Offshore Structure Congress–International Towing Tank Conference) joint group on uncertainties in wave modeling, discusses uncertainties associated with the long-term description of integrated wave parameters such as H_s and the zero-crossing/spectral wave period, which can have a significant impact on the extreme values used in design and marine operations. A review of probabilistic methods of extreme value assessment in hydro-climatological applications was presented by Nerantzaki & Papalexiou (2022).

Amarouche et al. (2023) investigated the sensitivity of extreme H_s estimates to the analyzed data sources and periods. Their analysis of extreme waves was performed with two different models, namely GEV distribution fitting to AM data and GP distribution fitting to POT data, and with four different wave data sources: (*a*) a 60-year wave hindcast data set developed with the Simulating Wave Nearshore (SWAN) model forced by Japanese 55-year Reanalysis (JRA-55) wind reanalysis, (*b*) the European Centre for Medium-Range Weather Forecasts (ECMWF) fifth-generation wave reanalysis (ERA5) for 40 years, (*c*) the satellite observations calibrated and provided by the Institut Français de Recherche pour l'Exploitation de la Mer (IFREMER) for 20 years, and (*d*) the wave buoy measurements provided by Copernicus for 40 and 30 years. The authors stated that the extreme wave estimates can change by more than 20% over most of the global ocean depending on the used wave sources, and therefore the choice of suitable sources must be carefully considered. A new way to decluster wave data for POT modeling based on the extremal index was proposed by Oikonomou et al. (2020).

Jonathan et al. (2021) considered the estimation of return values in the presence of model uncertainty. Based on very large, representative samples of data, return values can be estimated with low bias and uncertainty using a variety of estimators. However, for small samples, return value estimation is problematic, and the performance of different return value estimators varies considerably. Towe et al. (2024) extended this analysis to the multivariate setting, and in particular to the estimation of associated values in the presence of model uncertainty.

Quadrado & Serafin (2024) investigated extreme total water levels occurring during the year along the US Atlantic coast and whether their individual components, such as waves, tides, and storm surge, vary across regions and during the year. Variations across storm seasons in these processes may have implications for how large-scale changes to the climate impact hazards along open sandy coastlines and influence the robustness of extrapolating rare events from model fits to small data sets. For a more general treatment of spatial and temporal effects, readers are directed to Section 5.

Nonstationary extreme value analysis allows us to determine the probability of exceedance of extreme sea states, taking into account trends in the time series of data at hand (see also the discussions in Section 6). De Leo et al. (2021) evaluated a popular nonstationary modification of the GEV of Equation 4 called N-GEV, where Equation 5 provides a time-dependent version of the location parameter:

$$\mu = \mu_0 + \mu_t t. \quad 5.$$

This increases the number of parameters to fit from 3 to 4. They analyzed the reliability of this N-GEV formulation of H_s and peak wave period T_p under the assumption of a linear trend for time series of AM H_s in the Mediterranean Sea. A methodology to assess the significance of the results of the nonstationary model employed was proposed. Results showed that the nonstationary N-GEV provides advantages over the stationary GEV only when all metrics consistently indicate the presence of a trend. Görmüş et al. (2022) evaluated extreme waves in the Mediterranean and the Black Sea. AM series and partial duration series of the H_s were used from the ERA5 data set. GEV, Gumbel, Weibull, log-normal, and GP models were used to predict H_s for 50-, 100-, and 500-year return periods.

Nonlinear wave-induced response of structures is often very sensitive for details of the wave excitation (height, period, steepness, breaking, etc.). In order to resolve such response including all flow details, it is usually required to apply high-fidelity modeling. Such models cannot be run for long enough to assess lifetime loads on marine structures in practical design situations. Most extreme value prediction methods for highly nonlinear structure response therefore rely on multi-fidelity wave modeling, where only critical sea states or critical wave events are run with high-fidelity methods.

A complicating factor is that the largest ship response does not necessarily occur in the highest wave; the selection of critical sea states and wave events may also have to consider, e.g., wave steepness, wave direction, ship motions, wave breaking, etc. A review of suitable extreme value prediction methods for nonlinear waves and wave impact loads is provided by van Essen & Seyffert (2023). These methods include response-conditioning (or design wave) methods, screening methods, and adaptive sampling methods. In the first category, response-conditioning methods, Seyffert (2022) applied the NL-DLG (Nonlinear Design Loads Generator) design wave method to generate an ensemble of wave profiles, conditioned to contain rare wave groups. It can also be applied to other nonlinear responses to efficiently generate critical wave profiles for these responses. Most available design wave methods generate linear Gaussian wave profiles, which are not always sufficient for highly nonlinear structure response. Kim et al. (2022) defined a new design wave method, which can generate higher-order wave events by incorporating a HOS model.

The interest in applying adaptive sampling methods to wave-induced response increased after the publications of Mohamad & Sapsis (2018) and Gramstad et al. (2020). Guth & Sapsis (2022) applied adaptive sampling combined with Gaussian process surrogate regression in order to generate design wave groups (consisting of a set of Karhunen–Loève wave components) for nonlinear ship response. This can be seen as a combined response-conditioning and adaptive sampling method. The method was also applied to obtain wave loads on a monopile by Guth et al. (2024), showing that high-fidelity simulation of the resulting critical wave groups leads to reasonable response distributions (but has a tendency to underestimate the tails). A possible reason is that the number of considered wave components in the design waves has to be maximized for the method to be efficient. One way to overcome this is to combine screening with adaptive sampling instead. This way, the critical wave groups are not synthesized based on input conditions but are instead selected (screened) from a long lower-fidelity simulation. van Essen et al. (2023b) defined a new method that combines screening with (multi-fidelity) Gaussian process regression (GPR) and adaptive sampling. This pilot study also showed that the method could work for a simplified problem that selects critical linear wave events to predict the full second-order wave crest height distribution. Classical GPR constructs a surrogate for a limited number of high-fidelity data points $\mathbf{y}_h$. Multi-fidelity GPR also leverages low-fidelity information, for instance, using the linear autoregressive function in Equation 6 (Kennedy & O'Hagan 2000):

$$\mathbf{y}_h = \rho \mathbf{y}_l + \boldsymbol{\delta}. \quad 6.$$

Separate surrogates are constructed for the low-fidelity function $\mathbf{y}_l$ and for the difference function $\boldsymbol{\delta}$ that models the difference between the low- and high-fidelity information.

One drawback with most multi-fidelity extreme value prediction methods is the necessity to define input for high-fidelity wave modeling based on selected or generated wave events with a lower fidelity. Some ideas to reduce this problem are to use event matching procedures such as those described by Gramstad et al. (2023); to use coarse mesh computational fluid dynamics as a low-fidelity screening tool as demonstrated by, e.g., van Essen et al. (2021); or to use direct coupling methods as done by, e.g., Kamath et al. (2023).

The review article by Belzile et al. (2023) surveyed recent developments in software implementations for extreme value analyses potentially useful to the metocean practitioner. The open-source covXtreme software of Towe et al. (2024) provides functionality for estimation of marginal and conditional extreme value models, nonstationary with respect to covariates, and environmental design contours (see Section 4), tailored for metocean applications. The review by van Essen et al. (2026) contains a benchmark of available open-source implementations of extreme value prediction methods.

4. MULTIVARIATE ANALYSIS AND JOINT DISTRIBUTIONS

Reliability-based design of offshore structures usually requires joint probabilistic models to describe the long-term environmental scenario. Traditionally, wave scatter diagrams have been used for this purpose, but more information can be obtained by more careful joint models of the relevant environmental variables. The full probabilistic modeling of wind and wave parameters for a site in the South China Sea usually attacked by typhoons was studied by Song et al. (2022) (see also Song et al. 2024). The full probabilistic model of the environmental variables was developed using the C-vine copula method. An alternative joint model based on generative adversarial networks was proposed by Song et al. (2023). The application of the full probabilistic model to the fatigue analysis of a floating offshore wind turbine was illustrated through an example. Simão et al. (2022) presented a multi-dimensional joint model alternative for the probabilistic long-term environmental data description, including directional variables. The joint model aims to statistically describe the main environmental parameters for the probabilistic design of marine structures, such as floating production, storage, and offloading vessels systems. Liao et al. (2022) focused on the statistical characteristics of directional wave climate in the seasonal ice zone of the Barents Sea. The joint distributions of H_s, mean wave period, and mean wave direction were constructed using a mixture trivariate distribution model. The ocean data were classified into four groups based on the relative weights of the energy content of wind, wave, and swell fields. Yang et al. (2023) focused on the bivariate distribution of wind speed and air density with a mixture copula model, each component of which was constructed using a Weibull distribution for the wind speed, a log-normal distribution for the air density, and a Gaussian copula function for the description of the dependency structure. The optimum component number and maximum likelihood estimate of the mixture model were determined using the Bayesian information criterion and expectation–maximization algorithm, respectively. Copulas and Bayesian inference were applied to model H_s and T_p by Duan et al. (2024). Wang et al. (2023) proposed joint distribution of wind speed and direction over complex terrains based on nonparametric copula models.

Nonparametric approaches to joint distribution modeling have become popular in recent years (Han et al. 2018). Nonparametric kernel density estimators were combined by copulas to form a multivariate model of wave and wind conditions by Wen et al. (2024). Nonparametric marginals combined with nonparametric copulas for the dependence structure were proposed by Latif & Simonovic (2022). However, as shown by Vanem et al. (2024c), nonparametric models may fail to capture the tails of the distributions well, so some caution is advised if such models are to be used for extreme value analysis. A similar conclusion was reached by Meng & Li (2024).

The uncertainty of design parameters for marine structures based on multivariate models was analyzed by Liu et al. (2023a), who suggest that such uncertainty analysis should be considered during model selection. A joint probabilistic approach was presented by García-Rojo (2004) for establishing joint distributions of environmental variables from combined information from different measurement locations. The approach has been found to compare well with the commonly used measure-correlate-predict method.

The extremes of multiple variables are difficult to assess, and even the definition of a multivariate extreme value is ambiguous. Recent reviews of multivariate extreme value modeling were given by, e.g., Engelke & Ivanovs (2021) and Nolan (2024).

Environmental contours are a pragmatic and widespread method to estimate the long-term extreme response of marine structures. It is essentially a way to describe multi-variable extreme environmental conditions. Over the years, a range of approaches have been proposed. A benchmarking study was recently conducted by Haselsteiner et al. (2021) to compare the various methods using a common set of data. Nine teams of researchers contributed to the benchmark. The analysis of the submitted contours highlighted significant differences between contours derived via different methods. de Hauteclocque et al. (2022) extended this benchmark study by providing a quantitative assessment of the contours submitted to the exercise. Three main reasons for the discrepancies were identified: first, the statistical model used to model the joint probability; second, the effect of serial correlation, which is generally neglected; and last, the varying assumptions made by the different environmental contour methods. In recent years, progress has been made on each of those items.

A promising approach for modeling the extreme joint distribution, called SPAR (semi-parametric angular-radial), was introduced by Mackay et al. (2025), and implementation details and practical examples were provided by Murphy-Barltrop et al. (2024a). The approach relies on a transformation of variables to polar coordinates. For the general joint density function $f_{X,Y}$ of variables X and Y in Equation 7, this is formulated in Equation 8, where R is the radial variable, Θ is the angular variable, and $f_{R,\Theta}(r, \theta)$ is their joint density function:

$$f_{X,Y}(x,y) = f_X(x) f_{Y|X}(y|x), \quad 7.$$

$$R = \sqrt{(X^2 + Y^2)},$$

$$\Theta = \text{atan2}(X, Y), \text{ and} \quad 8.$$

$$f_{R,\Theta}(r,\theta) = r f_{X,Y}(r\cos\theta, r\sin\theta) = f_\Theta(\theta) f_{R|\Theta}(r|\theta).$$

The advantage of this transformation is that the density $f_{R,\Theta}$ for a given value of Θ is univariate (whereas $f_{X,Y}$ for a given value of Y is not). The tail of the radial variable is then modeled using a GP distribution, whose parameters are conditional on the angle. The inference can then be seen as a nonstationary POT analysis, where the angle is the covariate. Recently, the methodology has been extended into higher dimensions, with a deep learning approach used to estimate the dependence structure (Mackay et al. 2024). As the number of dimensions grows, the dependence structure becomes increasingly complex and computationally demanding to estimate. The use of a deep learning approach, whereby the threshold and GP parameter functions are represented using artificial neural networks, offers great flexibility in the dependence structures that can be represented, together with computationally efficient routines for training the model. This builds on a recent interest in the use of machine learning approaches for modeling the variation of extreme quantities, conditional on covariates (see, e.g., Pasche & Engelke 2024, Richards & Huser 2024).

On the serial correlation issue, discussed specifically by Mackay et al. (2021), an approach for partially accounting for serial dependence in the construction of environmental contours was proposed by Vanem (2023a,b) based on simulating a time series of a primary variable that preserves both the marginal distribution and auto-correlation structure. The approach discussed by Mackay & de Hauteclocque (2023), based on a collection of univariate fits, can also address this serial correlation issue. Moreover, by reducing the problem to a sequence of univariate analyses under various rotations of the coordinate axes, all the observations are used in the inference at each angle. This means that the method can be applied in an arbitrary number of dimensions, without increasing the sparsity of the data—a problem known as the curse of dimensionality. Finally, about the different assumptions of the environmental contour, Mackay & Haselsteiner (2021) clarified the distinction between contours defined in terms of marginal probabilities under rotations of the axes [such as IFORM (inverse first-order reliability method) contours and variants thereof] and contours defined in terms of the probability that an observation falls anywhere outside the contour (referred to as the total exceedance probability of the contour). Approaches like highest density contours (Haselsteiner et al. 2017) and ISORM (inverse second-order reliability method) (Chai & Leira 2018) are defined in terms of the total exceedance probability and are much more conservative than IFORM-type contours. This tends to result in an overestimate of extreme structural responses derived from these types of contours. Furthermore, the overestimation bias increases dramatically with the number of dimensions. This explains why the original IFORM and its derivatives (direct sampling and direct IFORM) are preferred in practical applications.

Other aspects of environmental contours have also been investigated in recent years. Huseby et al. (2021) improved on the direct sampling approach of Huseby et al. (2013, 2015) by introducing a more robust approach to extract the contour, with a more rigorous mathematical foundation. Closely related, Hafver et al. (2022) ingeniously observed that direct sampling contours can be extracted using Voronoi cells. Further to this finding, Mackay & de Hauteclocque (2023) combined this with the direct IFORM method (Derbanne & de Hauteclocque 2019) and proposed a robust way to derive contours in high numbers of dimensions, with an application in four dimensions. Environmental contours for multivariate extremes were also discussed by Simpson & Tawn (2025), involving a transformation to Laplace space and a definition of contours in radial-angular coordinates. Three-dimensional environmental contours accounting for sampling methods and season were presented by Meng & Li (2024), and three-dimensional contours based on direct sampling were discussed by Vanem (2019). Three-dimensional contours of wave height, current speed, and relative direction were presented by Mackay & Hardwick (2022), who showed that in locations with strong tidal flows, extreme wave heights tend to occur simultaneously with strong opposing currents.

For analyzing the uncertainty related to the construction of environmental contours, Zhao & Dong (2023b) assessed the uncertainty on the extreme mooring loads of floating systems considering short-term variability, including significant differences due to marginal distribution fitting, parameter estimation methods, and joint models. The metocean conditions conditioned on extreme structural responses were studied by Speers et al. (2024), who reported that environmental contours might not be conservative in every situation and suggested that contours may need to be calibrated for the particular structural response being analyzed in order to reduce bias. Similar results were reported by Seyffert & Kana (2020) and Haselsteiner et al. (2022). The environmental contour approach to assess extreme structural response was compared with a sequential sampling approach (Gramstad et al. 2020) by Wang et al. (2024), suggesting that the latter might be preferred in certain situations where the assumptions implicit in the environmental contour method are violated. An alternative approach based on scenario optimization was presented by Crespo et al. (2024). An IFORM for long-term extreme response analysis was proposed by Wang et al. (2025) and compared with environmental contours. They showed that extreme responses tend to occur inside the environmental contours, indicating that design sea conditions may not lie on the contours.

Huseby et al. (2015), Hafver et al. (2022) , and Zhao & Dong (2021) investigated the effect of statistical models for constructing the bivariate distribution of metocean data on design loads and reliability assessment of offshore structures. The variabilities in short-term extreme response and the contributions of all sea states are often ignored, and this can cause unreliable results. Considering concerns over the sensitivity of the hydrodynamic performance of floating structures to wave data, Zhao & Dong (2022) evaluated the effect of environmental conditions on the short-term extreme response parameters and presented a full long-term analysis for the failure probability evaluation of floating structures. Vanem et al. (2023, 2024a) and Zhao & Dong (2023a) provided realistic multivariate models of environmental parameters to accurately describe the statistical characteristics of metoceanic conditions based on copula models and construct environmental contours for the reliability-based design of marine structures. An extension of the conditional extremes models (Heffernan & Tawn 2004) for bivariate mixture models was presented by Tendijck et al. (2023), where the extremal dependence between the variables might be a mixture of simpler bivariate distributions.

Recently, in the statistical extreme value literature, there has been interest in modeling multivariate extremes in terms of a so-called limit set. Under certain assumptions, sets of scaled observations can be shown to converge onto a limit set (Davis et al. 1988, Nolde & Wadsworth 2022). Inference is conducted on either standard exponential or Laplace margins, with a parametric or semiparametric model for the shape of the limit set (Majumder et al. 2023, Simpson & Tawn 2024, Wadsworth & Campbell 2024). An interesting development was proposed by Murphy-Barltrop et al. (2024b), who used a deep learning approach to estimate limit sets in higher dimensions and applied the method to a case study of estimating joint extremes of H_s at five locations in the North Sea.

5. SPATIAL AND TEMPORAL STATISTICS

Since both individual waves and sea states evolve in space and time, the simultaneous statistics of spatial and temporal behavior of individual waves and sea states are important in many practical applications. For example, the conclusions from the ExWaMar (extreme wave warning criteria for marine structures) project, concerned with developing warning criteria for rogue waves (Bitner-Gregersen et al. 2024), emphasized that space-time data should be employed in the development of rogue wave warning criteria, both in order to reduce the sampling variability of measurements and because point crest statistics underestimate the statistics of crests over an area small enough to be relevant for ships and offshore structures. Because of the reduced costs and increased availability of dedicated open-source software, stereo-video imagery is increasingly used for wave measurements, and it is necessary to employ spatio-temporal techniques to analyze the data. Likewise, with the availability of inexpensive free-drifting wave buoys, the estimation of sea-state parameters based on a distributed set of buoy measurements requires analysis in time and space. A description of sea-state parameters in space and time can also be important in quantifying fatigue and extreme loads on ships in transit. In addition, there is an increased interest in large floating structures, such as energy storage hubs, floating solar panels, and even islands and airports. Such installations span larger areas than traditional marine structures, requiring more emphasis on space-time wave statistics.

5.1. Spatial and Temporal Statistics of Sea-State Parameters

Smit et al. (2021) used hourly H_s data provided by approximately 60 free-drifting directional wave buoys distributed in the northern Pacific to investigate the effect on wave forecast quality of using data assimilation techniques. Using the WAVEWATCH III wave model and a sequential optimal data assimilation technique, the accuracy of the wave forecast was investigated and compared with the forecast accuracy without using data assimilation. It was demonstrated that the use of a simple assimilation technique gave a 27% reduction in the root mean square error of the H_s forecast compared with the case where no data assimilation was used.

Building on the methodology for conditional extremes developed by Heffernan & Tawn (2004), Shooter et al. (2021, 2022) developed spatial conditional extreme models for storms. Shooter et al. (2021) considered storm peak H_s in the North Sea characterized by direction and distance and found reasonable agreement with hindcast results. Shooter et al. (2022) developed a multivariate spatial conditional extreme (MSCE) model suitable for investigating the characteristics of joint sea-state parameters given the occurrence of an extreme value of one of the parameters at one location. A joint model was conditioned on extreme satellite wind speed and developed for satellite wind speed, hindcast wind speed, and hindcast H_s over a satellite trajectory in the North Atlantic. The authors concluded that the spatial dependence of all three quantities decays over 600–800 km.

Nielsen (2022) used the ERA5 reanalysis database to investigate the variation in sea-state parameters for ships sailing on four typical ocean crossing routes. H_s, zero-crossing period, and mean wave direction were considered for different vessel speeds and different methods of interpolation of sea-state parameters. The study highlighted that the sea-state parameters vary rapidly along the ship route and that analysis of sea-state parameters should be based on bi-linear interpolation between the nearest grid points rather than the nearest neighbor approach.

Hildeman et al. (2021) used the stochastic partial differential equation approach in combination with a spatial deformation method to incorporate nonstationarity and anisotropy to model H_s. The model was fitted to the ERA-Interim hindcast data set for the North Atlantic, and the authors demonstrated how the model could be employed to estimate fatigue damage and wave height encountered for a ship in transit. Although the model does not consider the temporal evolution of H_s, this could be achieved by introducing a space-time separable covariance function.

Çelik (2022) and Altunkaynak et al. (2023) applied fuzzy logic techniques to the problem of short time forecasting of H_s and compared the results with several measurement stations in the North Pacific and North Atlantic. In order to improve the forecasting quality of machine learning techniques, it is useful to preprocess the data and attempt to sort the time series into stochastic and deterministic parts before developing the fuzzy logic algorithm. The authors have investigated preprocessing strategies based on the wavelet transform, singular value decomposition (Çelik 2022), and singular spectrum analysis (Altunkaynak et al. 2023) in addition to unprocessed data. They found that the singular spectrum analysis preprocessing algorithm was superior, with the ability to provide good estimates of H_s with a lead time of up to 12 hours over a range of water depths.

Whereas long-term analysis of extreme wave responses has traditionally been carried out by integrating all sea states consisting of blocks of piecewise constant sea-state parameters, long-term analysis of sea states is increasingly carried out for discrete storms where the evolution of the sea-state parameters during the storm is of interest. Tendijck et al. (2024a) used Markov processes to model extreme excursions in multivariate time series around the storm peak, introducing distinct peak, prepeak, and postpeak periods. For the postpeak period, for instance, an extremal vector autoregressive (EVAR) model for extremes of the d-dimensional process $[\mathbf{Y}_t = (Y_{t,1}, Y_{t,2}, \ldots, Y_{t,d})]$ with standard Laplace margins, for times $t = 1, 2, 3, \ldots$, combined the conditional extremes model of Heffernan & Tawn (2004) with a vector autoregressive model for the evolution of the time series at high levels. The EVAR model of order k with parameters $\mathbf{\Phi} \in \mathbb{R}^d \times \mathbb{R}^d$ for $\ell = 1, 2, \ldots, k$ and $\mathbf{B} \in (-\infty, 1)^d$ takes the form

$$\mathbf{Y}_{t+k}|(\mathbf{Y}_t, \ldots, \mathbf{Y}_{t+k-1}) = \sum_{\ell=1}^{k} \mathbf{\Phi}^{(\ell)} \mathbf{Y}_{t+k-\ell} + y^{\mathbf{B}} \boldsymbol{\epsilon}_t, \qquad 9.$$

assuming component-wise operations, with a local maximum of the first component $Y_{t,1} = y > u$ at time $t = 1$, where $u > 0$ is a large threshold. Furthermore, $\boldsymbol{\epsilon}_t$ is a d-dimensional multivariate random variable with nondegenerate margins, independent of $(Y_t, \ldots, Y_{t+k-1})$. Other details are given by Tendijck et al. (2024a). Using a simple structural response variable for a northern North Sea location, the model was shown to improve the description of extreme excursions in a storm compared with a historical storm matching approach.

A bivariate regional frequency analysis of H_s and wave periods was reported by Vanem (2020), 2021), extending previous applications of univariate regional frequency analyses in order to exploit spatial data in extreme value analysis (see also Bai et al. 2023). A spatial nonstationary extreme value analysis over the Mediterranean Sea was presented by De Leo et al. (2021).

A time-series model for H_s that preserves both the marginal distribution and the autocorrelation structure was proposed by Vanem (2023a,b), inspired by the work of Papalexiou (2018) and Papalexiou & Serinaldi (2020). This can account for the effect of serial correlation in the estimation of extreme values and can also be valuable in applications where the sequence of wave loadings is important. Time series of H_s were separated into deterministic and probabilistic components by Huang et al. (2024) to establish nonstationary time-series models accounting for climate variability.

Estimates of extreme environments and responses of offshore structures for tropical cyclone conditions are often subject to large uncertainties because of the short length of available time series for model estimation. Sando et al. (2024) proposed a methodology, building on the work of Wada et al. (2022) and earlier efforts, to characterize extreme multivariate time series for tropical cyclones, used to estimate return values of multivariate extremes from synthetic cyclone data for a spatial neighborhood of locations offshore from Guadeloupe (in the Lesser Antilles). Legrand et al. (2023) presented a model for the joint stochastic simulation of extreme coastal and offshore H_s.

Safe execution of offshore activities requires the forecasting of environmental time series to improve decision-making for, e.g., platform evacuation in severe weather, maintenance of offshore wind facilities, and on- and offloading from floating liquefied natural gas facilities. Good forecast performance for a range of environmental conditions is particularly important, as is reliable quantification of forecast uncertainty. Weather-forecasting organizations provide metocean forecasts to inform offshore activities, which can be calibrated further for use at specific locations, so that the calibrated forecast exhibits smaller bias and uncertainty than the original forecast. Moreover, modern forecasts tend to come in the form of a combination of different components, including, e.g., a deterministic forecast, a control forecast, and an ensemble of forecasts representing a range of possible future metocean temporal trajectories; all of these are available to calibrate the original forecast. There is a large literature on forecast calibration (for an introduction, see, e.g., Gneiting et al. 2005 and citations thereof). Bjerregard et al. (2021) and Allen et al. (2024) provided discussions of methods for achieving and assessing calibration of multivariate probabilistic forecasts, with Astfalck et al. (2023) focusing on metocean applications. Machine learning methods are increasingly employed for forecast calibration (e.g., Hoehlein et al. 2024, Tyralis & Papacharalampous 2024).

5.2. Spatial and Temporal Description of Individual Waves

Bitner-Gregersen & Gramstad (2021) studied the spatio-temporal sampling variability of the sea state recorded in the North Sea in 2018, where the "Justine Three Sisters" rogue waves were observed. Using a HOS model over a 3.5 × 3.5–km domain to third order in wave steepness, they ran 500 30-min simulations and extracted data from 256 evenly spaced points throughout the domain. They concluded that sampling variability over 30-min records in the statistics of individual waves is significant and should be considered carefully in design. The difference between statistics of crest height over an area and a point is large. They obtained a good fit to the distribution of surface elevation using Gram–Charlier series and recommended further investigations of the limitations and applications of the Gram–Charlier series.

Benetazzo et al. (2021) used stereo-imaging observations from a tropical storm in the Northwest Pacific to analyze the spatio-temporal extreme value statistics of maximum crests and wave heights. Using a consistent formulation for spatio-temporal extreme values based on the Tayfun and Bocotti (second-order/autocovariance) distributions for crest and wave height, respectively, they compared extreme values over several 20-min records over a 140 × 120–m area, with predictions based on spectral estimates from ECMWF. The spectra provided by ECMWF were compared with the spectra found from the stereo measurements. The authors found that the stereo-imaging results provided wave spectral parameters in good agreement with the ECMWF predictions. The largest wave heights and crests were located to the northeast of the eye of the storm, whereas the potential for rogue waves was found to the south/southwest. The agreement between measurements and predictions of extreme crests and wave heights is reasonable, although the authors acknowledged that the distributions used in the comparison are correct up to second order only.

In a subsequent analysis, Davison et al. (2022) used a HOS model to third order in wave steepness to simulate the same storm and compare with the stereo-imaging results. This study used spectral input from ECMWF at different stages in the storm, generating a large number of 20-min records of surface elevation, and studied the spatio-temporal extreme crest, focusing on the effect of crossing swell and wind seas. They found good agreement between the numerical results and measurements for both point statistics and area statistics. They found an increased space-time crest height probability for wind sea and swell systems crossing at 160 deg and proposed a consistent formulation for wave steepness in crossing seas, suggesting that it may be useful for predicting rogue wave–prone sea states.

In an analysis of large storm wave measurements in the eastern Mediterranean, Knobler et al. (2022) presented a novel analysis of waves in space and time, focusing on the potential risks posed by large waves. Using the Euler characteristics of Gaussian fields as a starting point and incorporating and modifying the third-order Tayfun–Fedele model for crest height at a fixed point, they proposed a formulation for crest height over an area, which took into account vessel area. In a striking example where forward speed is also taken into account, the authors demonstrated that the probability of encountering a critical crest height with probability 10^{-5} at a fixed point increased to 1/4,000 for a small vessel and 1/1,260 for a large vessel.

Malila et al. (2023) presented an analysis of 18 years of high-quality laser altimeter measurements from the central North Sea. This data set is complemented by stereo-video observations at the same location containing five individual storms collected in one winter season. The stereo-video data provide an opportunity to investigate the spatio-temporal evolution of the largest waves and compare with the laser point measurements. They observed a clear slowdown of the largest waves close to the position of maximum crest elevation. This is not found in linear or second-order theory but is consistent with higher-order nonlinear wave predictions and laboratory observations.

By investigating crest-area statistics with areas varying from zero to 60 × 60 m, they found differences between point and area crest height statistics up to a factor of 1.6, consistent with the linear and second-order crest-area predictions.

In order to obtain stable tail statistics of nonlinear wave properties in space and time, Tang & Adcock (2022a,b) have investigated methods for approximating the wave statistics using a limited number of deterministic wave groups calculated numerically using the modified nonlinear Schrodinger (MNLS) equation. By conditioning the initial conditions of the MNLS simulations, they concluded that this approach can provide accurate space-time statistics of wave properties at a fraction of the computational cost of direct nonlinear Monte Carlo simulations.

Tang & Adcock (2021) investigated the usefulness of data-driven methods for estimating crest distributions over space and time. Two simple machine learning approaches were employed to link sea-state parameters to crest distributions: a simple fit to Gumbel parameters and a random forest approach. Data were generated using second-order theory and the results were compared with established crest distribution models. The authors concluded that the data-driven approach could be useful for establishing crest distribution models; in particular, the random forest method performed well compared with the other methods.

6. NONSTATIONARY ANALYSIS AND COVARIATE EFFECTS

Environmental conditions will often be dependent on many factors, such as the season of the year, spatial location, long-term trends (e.g., due to climate change), and prevailing wind or wave directions. Thus, the assumption that the data are independent and identically distributed (iid) will generally not be fulfilled, and stationary models to describe the environment may, strictly speaking, not be correct if the conditions are nonstationary. These nonstationarities could be important, and statistical models incorporating them may represent a notable improvement compared with stationary models. The effect of nonstationarities would be important for both univariate and joint models, and for extreme value models and distribution models for all the data. However, it should be acknowledged that incorporating nonstationary effects in the statistical models introduces additional sources of uncertainty and that misspecification of the nonstationary parts of the model may give inconsistent results (Serinaldi & Kilsby 2015).

The causality in extremes of time series, where the extremes of one variable may be causally related to extremes in others with some delay, is discussed by Bodik et al. (2023). They introduced the causal tail coefficient for time series with extremal delay in order to describe such causal relationships and presented a case study on the influence of various phenomena on the weather and climate.

The importance of accounting for nonstationarity in risk and reliability assessments was discussed by Radfar & Galiatsatou (2023), who found that considering nonstationarity in extreme coastal events is important, and even more so when the dependence structure of the wave parameters is accurately modeled. Failure probabilities of coastal structures may be underestimated by up to 33% with stationary models that do not account for climatic trends. Similar results were found by Baldan et al. (2022), who reported that estimates of return levels of extreme sea levels are more conservative if nonstationary models are assumed. Obviously, the best model will be case specific, but at least for some applications, nonstationarities will be important and may influence inference about environmental conditions. A review of methods to detect, attribute, and manage nonstationarities in weather extremes was presented by Slater et al. (2021), who distinguished between two types of nonstationarities, trends and abrupt changes, and between different symptoms, such as nonstationarities in mean levels, variability, or frequency. They stressed that departure from stationarity should be detected and tested before engineering design is adjusted for such effects.

One way of accounting for nonstationarities is to include covariates (explanatory variables) in the statistical models. Another is to preprocess the data to remove the nonstationary effects. Yet another approach could be to use time series or spatial fields to account for autocorrelations and dependencies in space and time. One example of the latter is the statistical model for H_s presented by Vanem (2023b), where nonstationarities due to temporal dependencies at different timescales are modeled by first preprocessing the data to account for seasonal dependencies, and then a time-series model is established that preserves the short-term serial correlation. This was demonstrated to correct for the positive bias known to occur when ignoring serial correlation (Mackay et al. 2021). Different interpolation schemes to account for spatio-temporal variation of sea-state parameters along ship routes were presented by Nielsen (2022). However, in this subsection, the main focus is on statistical models accounting for nonstationary effects by adding covariates to the models.

A spatio-temporal model for H_s where wind fields are used as predictors was proposed by Obakrim et al. (2023). Both wind sea and swell are accounted for by combining a local predictor for the wind sea and a global predictor for the swell in a linear regression model. Hence, nonstationarities in space and time are modeled using wind field data as covariates. A vector autoregressive model with exogeneous input was used by Raudiya et al. (2021) for modeling and forecasting ocean waves based on wind information.

Seasonality is another source of nonstationarity that may be accounted for in the statistical models. This can be accounted for by preprocessing the data by estimating the seasonal effects and then fitting a stationary model on the residuals (see, e.g., Athanassoulis & Stefanakos 1995, Vanem 2018). Alternatively, nonstationary models accounting for the seasonality in different ways may be established. A rather cumbersome process for modeling seasonality in metocean data was proposed by Ma & Zhang (2024), where the data were segmented into short periods where the conditions were assumed to be stationary. Seasonality was modeled in extreme value models by D'Arcy et al. (2023), where skew-surges were modeled by a nonstationary GP distribution with daily covariates for the tails, and peak tides were allowed to vary over months and years. Dependence was accounted for by a tidal covariate, and temporal dependencies were adjusted for by the subasymptotic extremal index. Time series of H_s were decomposed into stationary and nonstationary components, where the stationary part was modeled as an autoregressive model and nonstationary components describing seasonality and trend were modeled by finite Fourier series expansion, by Nasir et al. (2023). Wind speed time series were decomposed into deterministic and stochastic components by Yang & Dong (2023), by combining signal decomposition methods and recurrence quantification analysis. This was reported to be able to capture complicated nonstationary patterns without relying on covariate information.

Metocean conditions are often associated with directions, and directional models are often useful in design. Hence, storm direction (mean wave direction) was included as a covariate in the nonstationary models proposed by Malliouri et al. (2023) for modeling coastal storms. They established a directional extreme value model where the parameters of the GP distributions were allowed to vary smoothly by direction, using a finite-order Fourier expansion. More generally, we express the values $\boldsymbol{\eta} \in \mathbb{R}^m$ of any extreme value model parameter (e.g., GP threshold, scale, or shape) at m index locations on the covariate domain in terms of linear combinations of q-dimensional basis functions (e.g., for Gaussian processes, splines, or Fourier series), represented by basis matrix $\boldsymbol{\Gamma} \in \mathbb{R}^m \times \mathbb{R}^q$. The task of the statistical inference then becomes optimal estimation of basis coefficients $\boldsymbol{\beta} \in \mathbb{R}^q$ such that

$$\boldsymbol{\eta} = \boldsymbol{\Gamma}\boldsymbol{\beta}. \qquad 10.$$

Further details were provided by Zanini et al. (2020). These and similar approaches can be used to obtain return values for H_s as a function of storm direction. An approach to jointly estimate the parameters of frequency–direction wave spectra from three-dimensional buoy time series was outlined by Grainger et al. (2023), based on a multivariate extension of the debiased Whittle likelihood. This was reported to yield a significant improvement in performance compared with least-squares or moment-matching techniques.

Climate change and large-scale climate variability are drivers for nonstationarities in metocean data, and several papers deal with how to incorporate this into probabilistic models. Ewans & Jonathan (2023) presented an investigation into the uncertainties in estimating the effect of climate change on the 100-year return values of H_s using nonstationary extreme value models. They reported that there is large variability in return value estimates from different climate models (general circulation models) and that these sources of uncertainty seem to be larger than typical modeling choices for extreme value models. They reported variations in return value estimates of $\pm 15\%$; Leach et al. (2025) provided a related study. The influence of climate variability on ocean wave energy was investigated by Sardana et al. (2024). A nonstationary model with different climate variability indices as covariates was assumed, indicating that both the mean and extreme wind sea and swell wave energy are influenced by the different climate modes (El Niño–Southern Oscillation, North Atlantic Oscillation, Indian Ocean Dipole, and Southern Annular Mode). These studies highlight the importance of accounting for climate change and climate variability in statistical models of the ocean environment. IOGP (2024) provides guidance on potential effects of climate change on metocean design and operating criteria.

Four different approaches to model the influence of climate variability on global ocean waves were explored by Liu et al. (2023b): linear regression, composite analysis, empirical orthogonal functions (EOF), and wavelet analysis. The linear regression model uses six common climate oscillations as regressors for ocean waves. The composite analysis determines the mean values of the wave parameters for years when the climate index anomalies are outside ± 1 standard deviation. EOF analysis decomposes the data into spatial and temporal patterns and calculates the correlation between the principal components and the climate indices. Wavelet analysis decomposes the data into the time-frequency domain to identify the dominant periods and how they evolve over time, and to analyze the correlation between the wave climate and the climate index anomalies. Other climate variability indices were used as covariates in nonstationary GEV distributions for extreme H_s and wind speeds over the Indo-Pacific Ocean by Kumar et al. (2024). Li et al. (2024) presented nonstationary models combining a Poisson distribution for the frequency of typhoons, and Gumbel, Weibull, and GP distributions for H_s. Varying typhoon frequencies were modeled by different Poisson rates for different time periods, and nonstationarities were modeled by assuming time-dependent linear functions for the location parameter in the models for H_s. They reported a notable difference between return value estimates obtained from stationary models. Nonstationary GP distribution models were also presented by D'Arcy et al. (2022) for skew-surges, with covariates accounting for both climate change and seasonality.

Nonstationarities may have a particular impact on estimates of extreme events, and several recent papers have reported applications of nonstationary extreme value analysis for metocean data. A recent literature review of applications of nonstationary extreme value analysis in coastal regions was presented by Radfar et al. (2023). Arif et al. (2022) introduced nonstationary GEV models to model extreme wind loads, where the GEV model parameters are modeled by different functions of time, similar to the models of Vanem (2015) (see also De Leo et al. 2021). A nonlinear POT model was proposed by Barlow et al. (2023) as a piecewise-linear model in one- or two-dimensional covariates related to season and direction. Another seasonal-directional extreme value model for individual wave heights was proposed by Bohlinger et al. (2023), where cyclic cubic splines are used to model the effect of varying season and direction. A multivariate (wind speed and H_s) spatial conditional extremes model was presented by Shooter et al. (2022), involving nonstationary directional-seasonal marginal extreme value analyses at specific locations and extremal spatial dependence between the locations.

As this brief literature survey suggests, several applications of nonstationary modeling of metocean variables have been described. Typically, nonstationarities are related to spatio-temporal variabilities at different scales: seasonality, directionality, climate variability, and trends due to climate change. A particular focus of several studies is on the nonstationarities of extremes. Several methods exist for taking such effects into account, and the preferred approach will be highly case specific. These methods should be considered in cases where departure from stationarity is deemed to be important and the iid assumption cannot easily be defended. Tendijck et al. (2024b) performed a simulation study, the results of which emphasize the critical role of tuning parameter settings in practical nonstationary extreme value analysis of POT using the GP distribution.

7. SUMMARY

This article has presented a review of recent developments in statistical modeling of the environment relevant for the design and operation of ship structures. The increasing access to relevant environmental data from observations, numerical models and hindcasts, and laboratory experiments, with increasing resolution, spatial and temporal coverage, and accuracy, combined with increasing computational capabilities, makes more complicated statistical models possible. For example, nonstationary models that can take covariate effects into account have become more widespread, and several new statistical models have been proposed, including temporal, spatial, and spatio-temporal models. Moreover, joint models for the distribution of several metocean variables have been proposed, enabling probabilistic descriptions of the environment with increasing dimensionality. Another trend is in incorporating and propagating different sources of uncertainty, both aleatory and epistemic, throughout the modeling chain—for example, by Bayesian methods. Much research has also focused on improved modeling of short-term and long-term statistics of sea states and individual waves, with special attention to the effect of varying water depths. Several new models have been proposed especially for shallow-water conditions relevant for coastal areas. Another trend is the proposal of nonparametric models in the literature. These offer flexible alternatives to parametric models that can fit arbitrary patterns in a sample well. However, generalizability can be an issue with such approaches, and it is generally advised to use such models with care, especially if the purpose is to study extremes and there is a need for extrapolation. Finally, several new models for extreme value analysis have been proposed for both univariate and multivariate problems. In particular, the semiparametric angular-radial approach to multivariate extremes is believed to be an interesting approach, and it is expected that practical experience with such models in design and operation will provide further insight into the usefulness of this approach.

We note that several new approaches to utilize the statistical models to account for the environment in design and operation of ships and other marine structures have been proposed. For example, a traditional design approach is to identify a design wave or design conditions and make sure the structure is designed to withstand such conditions. This approach implicitly assumes that the most critical response occurs in the most severe environmental conditions, and it does not take short-term variability into account. New approaches that combine long-term and short-term variability, e.g., based on adaptive sampling and surrogate models, challenge this assumption and are believed to represent important developments (Gramstad et al. 2020, Speers et al. 2024). However, even though such developments are relevant for the application of statistical models of the environment, this topic is considered outside of scope of this article on statistical modeling.

SUMMARY POINTS

1. Recent trends toward increasing availability of data and increasing computational capabilities have enabled more complicated statistical modeling of the ocean environment.
2. Nonstationary models accounting for covariate effects have been developed, describing temporal, spatial, and directional effects on the ocean environment, as well as the effects of long-term trends due to climate change.
3. Because ocean structures are simultaneously subjected to wind, waves, and other marine phenomena, joint probability models of environmental variables have been developed in increasingly higher dimensions, utilizing both copulas and other parametric and nonparametric methods.
4. Reliable predictions of extreme values have a direct impact on the design and operation of marine structures and may influence the safety and cost of marine structures. Much focus has therefore been put on the modeling of extreme weather events at sea. This includes new approaches to multivariate extremes.
5. Different sources of uncertainty, including both aleatory and epistemic uncertainties, are increasingly being incorporated into the statistical models of the ocean environment and propagated through the modeling chain. Bayesian methods are useful tools for handling this.
6. Much research has focused on the short-term statistics of wave and crest heights in unidirectional regular and irregular sea states, as well as directionally spread sea states, with a particular focus on the effects of varying water depths and shallow-water conditions.

FUTURE ISSUES

1. We anticipate that statistical modeling of the ocean environment will remain an active area of research in the near future, partly continuing along the same directions as recent developments.
2. Many renewable energy facilities are being developed in coastal regions prone to tropical cyclones (hurricanes and typhoons) around the world. Hence, we anticipate increased attention on reliable statistical modeling of tropical cyclones, leading to improved short- and long-term statistical models for extreme wave and wind conditions with finer spatial and temporal resolution.
3. Continued focus on the effect of climate change and considerations of nonstationary random processes and joint distributions is likely, which is of great significance for evaluating the long-term effects of the ocean environment on marine structures and structural safety.
4. Data-driven models utilizing machine learning and artificial intelligence will be increasingly used for extreme weather prediction and description. However, the lack of physical interpretation needs to be remedied by physics-informed and gray-box models (e.g., physics informed neural networks). Explainable artificial intelligence will be an important area of research.

5. Increased use of synthetic data is predicted, to supplement the limited spatial and temporal coverage of available observations and hindcast data.

DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS

This article is partly based on the work of the technical committee I.1 Environment of the International Ship and Offshore Structures Congress (ISSC) 2025 (van Essen et al. 2026).

LITERATURE CITED

Allen S, Ziegel J, Ginsbourger D. 2024. Assessing the calibration of multivariate probabilistic forecasts. *Q. J. R. Meteorol. Soc.* 150:1315–35

Altunkaynak A, Çelik A, Mandev MB. 2023. Hourly significant wave height prediction via singular spectrum analysis and wavelet transform based models. *Ocean Eng.* 281:114771

Amarouche K, Akpnar A, Kamranzad B, Khames GEY. 2023. Global extreme wave estimates and their sensitivity to the analysed data period and data sources. *Mar. Struct.* 92:103494

Arif M, Khan F, Ahmed S, Imtiaz S. 2022. Extreme wind load analysis using non-stationary risk-based approach. *Saf. Extreme Environ.* 4(3):247–55

Astfalck L, Bertolacci M, Cripps E. 2023. Evaluating probabilistic forecasts for maritime engineering operations. *Data-Centric Eng.* 4:e15

Athanassoulis GA, Stefanakos CN. 1995. A nonstationary stochastic model for long-term time series of significant wave height. *J. Geophys. Res. Oceans* 100(C8):16149–62

Babanin AV, Bernardino M, von Bock und Polach F, Campos R, Ding J, et al. 2022. Committee I.1: environment. Paper presented at the 21st International Ship and Offshore Structures Congress, Sept. 11–15

Bai G, Ruan Z, Wang J. 2023. The bivariate region frequency extreme value analysis of significant wave height and mean wave period in the South China Sea. *Ocean Eng.* 276:114151

Baldan D, Coraci E, Crosato F, Ferla M, Bonometto A, Morucci S. 2022. Importance of non-stationary analysis for assessing extreme sea levels under sea level rise. *Nat. Hazards Earth Syst. Sci.* 22(11):3663–77

Barlow AM, Mackay E, Eastoe E, Jonathan P. 2023. A penalised piecewise-linear model for non-stationary extreme value analysis of peaks over threshold. *Ocean Eng.* 267:113265

Belzile LR, Dutang C, Northrop PJ, Opitz T. 2023. A modeler's guide to extreme value software. *Extremes* 26:595–638

Benetazzo A, Barbariol F, Bergamasco F, Bertotti L, Yoo J, et al. 2021. On the extreme value statistics of spatio-temporal maximum sea waves under cyclone winds. *Progress Oceanogr.* 197:102642

Bitner-Gregersen EM, Gramstad O. 2021. Statistical description of nonlinear waves. In *Proceedings of the ASME 2021 40th International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 2: *Structures, Safety, and Reliability*. American Society of Mechanical Engineers

Bitner-Gregersen EM, Gramstad O, Trulsen K, Magnusson AK, Støle-Hentschel S, et al. 2024. Rogue waves: results of the ExWaMar project. *Ocean Eng.* 292:116543

Bitner-Gregersen EM, Waseda T, Parunov J, Yim S, Hirdaris S, et al. 2022. Uncertainties in long-term wave modelling. *Mar. Struct.* 84:103217

Bjerregard MB, Moeller JK, Madsen H. 2021. An introduction to multivariate probabilistic forecast evaluation. *Energy AI* 4:100058

Bodik J, Palu M, Pawlas Z. 2023. Causality in extremes of time series. *Extremes* 27:67–121

Bohlinger P, Economou T, Aarnes OJ, Malila M, Breivik Ø. 2023. A general framework to obtain seamless seasonal–directional extreme individual wave heights—showcase Ekofisk. *Ocean Eng.* 270:113535

Bonar PA, Fitzgerald CJ, Lin Z, van den Bremer TS, Adcock TA, Borthwick AG. 2021. Anomalous wave statistics following sudden depth transitions: application of an alternative Boussinesq-type formulation. *J. Ocean Eng. Mar. Energy* 7:145–55

Çelik A. 2022. Improving prediction performance of significant wave height via hybrid SVD-fuzzy model. *Ocean Eng.* 266:113173

Chai W, Leira BJ. 2018. Environmental contours based on inverse SORM. *Mar. Struct.* 60:34–51

Crespo LG, Agrell C, Vanem E. 2024. A scenario optimization approach to the prediction of extreme structural responses to environmental loading. In *Proceedings of the ASME 2024 43rd International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 2: *Structures, Safety, and Reliability*. American Society of Mechanical Engineers

D'Arcy E, Tawn JA, Joly A, Sifnioti DE. 2023. Accounting for seasonality in extreme sea-level estimation. *Ann. Appl. Stat.* 17(4):3500–25

D'Arcy E, Tawn JA, Sifnioti DE. 2022. Accounting for climate change in extreme sea level estimation. *Water* 14(19):2956

Davis RA, Mulrow E, Resnick SI. 1988. Almost sure limit sets of random samples in $\mathbf{R}^d$. *Adv. Appl. Probab.* 20(3):573–99

Davison S, Benetazzo A, Barbariol F, Ducrozet G, Yoo J, Marani M. 2022. Space-time statistics of extreme ocean waves in crossing sea states. *Front. Mar. Sci.* 9:1002806

de Hauteclocque G, Mackay E, Vanem E. 2022. Quantitative comparison of environmental contour approaches. *Ocean Eng.* 245:110374

de Hauteclocque G, Maretic NV, Derbanne Q. 2023. Hindcast based global wave statistics. *Appl. Ocean Res.* 130:103438

De Leo F, Besio G, Briganti R, Vanem E. 2021. Non-stationary extreme value analysis of sea states based on linear trends. Analysis of annual maxima series of significant wave height and peak period in the Mediterranean Sea. *Coast. Eng.* 167:103896

Derbanne Q, de Hauteclocque G. 2019. A new approach for environmental contour and multivariate de-clustering. In *Proceedings of the ASME 2019 38th International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 3: *Structures, Safety, and Reliability*. American Society of Mechanical Engineers

Duan X, Wang S, Liu D, Shi J, Wu Y, Zhou X. 2024. A statistical analysis method for significant wave height and spectral peak frequency considering the random and time-varying effects based on copula function and Bayesian inference. *Ocean Model.* 190:102390

Engelke S, Ivanovs J. 2021. Sparse structures for multivariate extremes. *Annu. Rev. Stat. Appl.* 8:241–70

Ewans K, Jonathan P. 2023. Uncertainties in estimating the effect of climate change on 100-year return value for significant wave height. *Ocean Eng.* 272:113840

Fuhrman DR, Klahn M, Zhai Y. 2023. A new probability density function for the surface elevation in irregular seas. *J. Fluid Mech.* 970:A38

García-Rojo R. 2004. Algorithm for the estimation of the long-term wind climate at a meteorological mast using a joint probabilistic approach. *Wind Eng.* 28(2):213–23

Gneiting T, Raftery AE, Westveld AH, Goldman T. 2005. Calibrated probabilistic forecasting using ensemble model output statistics and minimum CRPS estimation. *Mon. Weather Rev.* 133:1098–118

Görmüş T, Ayat B, Aydoğan B. 2022. Statistical models for extreme waves: comparison of distributions and Monte Carlo simulation of uncertainty. *Ocean Eng.* 248:110820

Gouin M, Ducrozet G, Ferrant P. 2016. Development and validation of a non-linear spectral model for water waves over variable depth. *Eur. J. Mech. B Fluids* 57:115–28

Grainger JP, Sykulski AM, Ewans K, Hansen HF, Jonathan P. 2023. A multivariate pseudo-likelihood approach to estimating directional ocean wave models. *J. R. Stat. Soc. Ser. C* 72(3):544–65

Gramstad O, Agrell C, Bitner-Gregersen E, Guo B, Ruth E, Vanem E. 2020. Sequential sampling method using Gaussian process regression for estimating extreme structural response. *Mar. Struct.* 72:102780

Gramstad O, Johannessen TB, Lian G. 2023. Long-term analysis of wave-induced loads using High Order Spectral Method and direct sampling of extreme wave events. *Mar. Struct.* 91:103473

Gramstad O, Lian G. 2024. Parametrization of sea surface skewness and kurtosis with application to crest distributions. *J. Fluid Mech.* 979:A4
Guth S, Katsidoniotaki E, Sapsis TP. 2024. Statistical modeling of fully nonlinear hydrodynamic loads on offshore wind turbine monopile foundations using wave episodes and targeted CFD simulations through active sampling. *Wind Energy* 27(1):75–100
Guth S, Sapsis TP. 2022. Wave episode based Gaussian process regression for extreme event statistics in ship dynamics: between the Scylla of Karhunen–Loève convergence and the Charybdis of transient features. *Ocean Eng.* 266:112633
Hafver A, Agrell C, Vanem E. 2022. Environmental contours as Voronoi cells. *Extremes* 25(3):451–86
Han Q, Hao Z, Hu T, Chu F. 2018. Non-parametric models for joint probabilistic distributions of wind speed and direction data. *Renew. Energy* 126:1032–42
Haselsteiner AF, Coe RG, Manuel L, Chai W, Leira B, et al. 2021. A benchmarking exercise for environmental contours. *Ocean Eng.* 236:109504
Haselsteiner AF, Frieling M, Mackay E, Sander A, Thoben KD. 2022. Long-term extreme response of an offshore turbine: How accurate are contour-based estimates? *Renew. Energy* 181:945–65
Haselsteiner AF, Ohlendorf JH, Wosniok W, Thoben KD. 2017. Deriving environmental contours from highest density regions. *Coast. Eng.* 123:42–51
Haver S, Winterstein SR. 2008. Environmental contour lines: a method for estimating long term extremes by a short term analysis. Paper presented at the SNAME Maritime Convention, Oct. 17–18
Heffernan JE, Tawn JA. 2004. A conditional approach for multivariate extreme values (with discussion). *J. R. Stat. Soc. Ser. B* 66(3):497–546
Hildeman A, Bolin D, Rychlik I. 2021. Deformed SPDE models with an application to spatial modeling of significant wave height. *Spat. Stat.* 42:100449
Hoehlein K, Schulz B, Westermann R, Lerch S. 2024. Postprocessing of ensemble weather forecasts using permutation-invariant neural networks. *Artif. Intell. Earth Syst.* 3:e230070
Hossain MN, Araki S, Hoque A, Josiah N. 2024. Wave height distribution for plunging breakers induced by air bubbles. *Ocean Eng.* 309:118472
Hossain MN, Rahman M, Hoque A. 2022. Statistical distribution of wave heights attenuation by entrained air bubbles in the surf zone. *Ocean Eng.* 250:110911
Huang W, Zhu X, Jin Y, Shen X. 2024. Nonstationary modelling of significant wave height using time series decomposition method. *Ocean Eng.* 310:118731
Huseby AB, Vanem E, Agrell C, Hafver A. 2021. Convex environmental contours. *Ocean Eng.* 235:109366
Huseby AB, Vanem E, Natvig B. 2013. A new approach to environmental contours for ocean engineering applications based on direct Monte Carlo simulations. *Ocean Eng.* 60:124–35
Huseby AB, Vanem E, Natvig B. 2015. Alternative environmental contours for structural reliability analysis. *Struct. Saf.* 54:32–45
IOGP (International Association of Oil and Gas Providers). 2024. *Potential climate change effects of metocean design and operating criteria*. Rep. 662, IOGP
Jonathan P, Randell D, Wadsworth J, Tawn J. 2021. Uncertainties in return values from extreme value analysis of peaks over threshold using the generalised Pareto distribution. 220:107725
Kamath A, Wang W, Pákozdi C, Bihs H. 2023. Identification and investigation of extreme events using an arbitrary Lagrangian–Eulerian approach with a Laplace equation solver and coupling to a Navier–Stokes solver. *J. Offshore Mech. Arct. Eng.* 145(6):061902
Karmpadakis I, Swan C. 2022. A new crest height distribution for nonlinear and breaking waves in varying water depths. *Ocean Eng.* 266:112972
Karmpadakis I, Swan C, Christou M. 2022. A new wave height distribution for intermediate and shallow water depths. *Coast. Eng.* 175:104130
Kennedy MC, O'Hagan A. 2000. Predicting the output from a complex computer code when fast approximations are available. *Biometrika* 87(1):1–13
Kim S, Bouscasse B, Ducrozet G, Canard M, Hauteclocque GD, et al. 2022. Numerical and experimental study of a FORM-based design wave applying the HOS-NWT nonlinear wave solver. *Ocean Eng.* 263:112287
Knobler S, Liberzon D, Fedele F. 2022. Large waves and navigation hazards of the Eastern Mediterranean Sea. *Sci. Rep.* 12(1):16511

Kumar P, Yadav A, Sardana D, Prasad R, Rajni. 2024. Extreme wave height response to climate modes and its association with tropical cyclones over the Indo-Pacific Ocean. *Ocean Eng.* 296:116789

Latif S, Simonovic SP. 2022. Nonparametric approach to copula estimation in compounding the joint impact of storm surge and rainfall events in coastal flood analysis. *Water Resourc. Manag.* 36(14):5599–632

Lawrence C, Gramstad KTO. 2020. Evolution of extreme wave statistics in surface elevation and velocity field over a non-uniform depth. *Phys. Fluids* 26:051705

Leach C, Ewans K, Jonathan P. 2025. Changes over time in the 100-year return value of climate model variables. *Ocean Eng.* 324:120605

Legrand J, Ailliot P, Naveau P, Raillard N. 2023. Joint stochastic simulation of extreme coastal and offshore significant wave heights. *Ann. Appl. Stat.* 17:3363–83

Li J, Li S, Li Y, Chen W, Li B, et al. 2024. Evaluations of extreme wave heights around Hainan Island and their uncertainty induced by decadal variations of input variables. *Ocean Eng.* 294:116705

Liao Z, Huang W, Dong S, Li H. 2022. Modelling trivariate distribution of directional ocean data in the Barents Sea seasonal ice zone. *Ocean Eng.* 260:111745

Liu G, Zhou X, Kou Y, Wu F, Zhao D, Xu Y. 2023a. Uncertainty analysis for the calculation of marine environmental design parameters in the South China Sea. *J. Oceanol. Limnol.* 41(2):427–43

Liu J, Meucci A, Young IR. 2023b. A comparison of multiple approaches to study the modulation of ocean waves due to climate variability. *J. Geophys. Res. Oceans* 128:e2023JC019843

Longuet-Higgins M. 1952. On the statistical distribution of the heights of sea waves. *J. Mar. Res.* 11(3):245–66

Ma P, Zhang Y. 2024. A time-varying copula approach for describing seasonality in multivariate ocean data. *Mar. Struct.* 94:103567

Mackay EBL, de Hauteclocque G. 2023. Model-free environmental contours in higher dimensions. *Ocean Eng.* 273:113959

Mackay EBL, de Hauteclocque G, Vanem E, Jonathan P. 2021. The effect of serial correlation in environmental conditions on estimates of extreme events. *Ocean Eng.* 242:110092

Mackay EBL, Hardwick JP. 2022. Joint extremes of waves and currents at tidal energy sites in the English Channel. In *Proceedings of the ASME 2022 41st International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 8: *Ocean Renewable Energy*. American Society of Mechanical Engineers

Mackay EBL, Haselsteiner AF. 2021. Marginal and total exceedance probabilities of environmental contours. *Mar. Struct.* 75:102863

Mackay EBL, Murphy-Barltrop C, Jonathan P. 2025. The SPAR model: a new paradigm for multivariate extremes: application to joint distributions of metocean variables. *J. Offshore Mech. Arct. Eng.* 147:011205

Mackay EBL, Murphy-Barltrop C, Richards J, Jonathan P. 2024. Deep learning joint extremes of metocean variables using the SPAR model. Preprint, arXiv:2412.15808 [stat.ML]

Majumder R, Shaby BA, Reich BJ, Cooley D. 2023. Semiparametric estimation of the shape of the limiting multivariate point cloud. Preprint, arXiv:2306.13257 [stat.ME]

Malila MP, Barbariol F, Benetazzo A, Breivik Ø, Magnusson AK, et al. 2023. Statistical and dynamical characteristics of extreme wave crests assessed with field measurements from the North Sea. *J. Phys. Oceanogr.* 53(2):509–31

Malliouri DI, Moraitis V, Petrakis S, Vandarakis D, Hatiris GA, Kapsimalis V. 2023. A non-stationary and directional probabilistic analysis of coastal storms in the Greek Seas. *Water* 15(13):2455

Mendez FJ, Losada IJ, Medina R. 2004. Transformation model of wave height distribution on planar beaches. *Coast. Eng.* 50(3):97–115

Meng X, Li ZX. 2024. 3-dimensional environmental contours of winds and waves accounting for different sampling methods and seasonal effects. *Ocean Eng.* 304:117724

Miche M. 1944. Mouvements ondulatoires de la mer en profondeur constante ou décroissante. *Ann. Ponts Chaussées* 114:25

Mohamad MA, Sapsis TP. 2018. Sequential sampling strategy for extreme event statistics in nonlinear dynamical systems. *PNAS* 115(44):11138–43

Murphy-Barltrop CJR, Mackay E, Jonathan P. 2024a. Inference for bivariate extremes via a semi-parametric angular-radial model. *Extremes* 28:209–38

Murphy-Barltrop CJR, Majumder R, Richards J. 2024b. Deep learning of multivariate extremes via a geometric representation. Preprint, arXiv:2406.19936 [stat.ME]

Nasir F, Taib CMIC, Ariffin EH, Padlee SF, Akhir MF, et al. 2023. Significant wave height modelling and simulation of the monsoon-influenced South China Sea coast. *Ocean Eng.* 277:114142

Nerantzaki SD, Papalexiou SM. 2022. Assessing extremes in hydroclimatology: a review on probabilistic methods. *J. Hydrol.* 605:127302

Nielsen UD. 2022. Spatio-temporal variation in sea state parameters along virtual ship route paths. *J. Oper. Oceanogr.* 15(3):169–86

Nolan JP. 2024. Modeling multivariate extremes. *WIREs Comput. Stat.* 16(2):e1652

Nolde N, Wadsworth JL. 2022. Linking representations for multivariate extremes via a limit set. *Adv. Appl. Probab.* 54(3):688–717

Obakrim S, Ailliot P, Monbet V, Raillard N. 2023. Statistical modeling of the space–time relation between wind and significant wave height. *Adv. Stat. Climatol. Meteorol. Oceanogr.* 9(1):67–81

Oikonomou C, Gradowski M, Kalogeri C, Sarmento A. 2020. On defining storm intervals: extreme wave analysis using extremal index inferencing of the run length parameter. *Ocean Eng.* 217:107988

Papalexiou SM. 2018. Unified theory for stochastic modelling of hydroclimatic processes: preserving marginal distributions, correlation structures, and intermittency. *Adv. Water Resourc.* 115:234–52

Papalexiou SM, Serinaldi F. 2020. Random fields simplified: preserving marginal distributions, correlations, and intermittency, with applications from rainfall to humidity. *Water Resourc. Res.* 56(2):e2019WR026331

Pasche OC, Engelke S. 2024. Neural networks for extreme quantile regression with an application to forecasting of flood risk. *Ann. Appl. Stat.* 18(4):2818–39

Petrova PG, Soares CG, Aguiar TC, Esperança PT. 2022. Statistical distributions of nonlinear waves from random laboratory wave fields. *Ocean Eng.* 243:110170

Qin J. 2022. Evolving probabilistic modeling for long-term significant wave heights with a focus on extremes. *Renew. Energy* 187:362–70

Quadrado GP, Serafin KA. 2024. The timing, magnitude, and relative composition of extreme total water levels vary seasonally along the U.S. Atlantic coast. *J. Geophys. Res. Oceans* 129(9):e2023JC020557

Radfar S, Galiatsatou P. 2023. Influence of nonstationarity and dependence of extreme wave parameters on the reliability assessment of coastal structures - a case study. *Ocean Eng.* 273:113862

Radfar S, Galiatsatou P, Wahl T. 2023. Application of nonstationary extreme value analysis in the coastal environment – a systematic literature review. *Weather Clim. Extremes* 41:100575

Ramezani M, Choe DE, Heydarpour K, Koo B. 2023. Uncertainty models for the structural design of floating offshore wind turbines: a review. *Renew. Sustain. Energy Rev.* 185:113610

Raudiya F, Rohmawati AA, Adytia D. 2021. Non-stationary order of vector autoregression in significant ocean wave forecasting. In *2021 9th International Conference on Information and Communication Technology (ICoICT)*. IEEE

Richards J, Huser R. 2024. Extreme quantile regression with deep learning. In *Handbook on Statistics of Extremes*, ed. M de Carvalho, R Huser, P Naveau, BJ Reich. Chapman and Hall/CRC

Sando K, Wada R, Rohmer J, Jonathan P. 2024. Multivariate spatial and spatio-temporal models for extreme tropical cyclone seas. *Ocean Eng.* 309:118365

Sardana D, Kumar P, Rajni. 2024. Influence of climate variability modes over wind-sea and swell generated wave energy. *Ocean Eng.* 291:116471

Scharnke J, van Essen SM, Seyffert HC. 2023. Required test durations for converged short-term wave and impact extreme value statistics—part 2: deck box dataset. *Mar. Struct.* 90:103411

Serinaldi F, Kilsby CG. 2015. Stationarity is undead: uncertainty dominates the distribution of extremes. *Adv. Water Resourc.* 77:17–36

Seyffert HC. 2022. Generating an ensemble of mutually exclusive and exhaustive waves targeted for extreme responses. *Ocean Eng.* 243:110172

Seyffert HC, Kana AA. 2020. Response-based reliability contours for complex marine systems considering short and long-term variability. *Appl. Ocean Res.* 103:102332

Shooter R, Ross E, Ribal A, Young IR, Jonathan P. 2022. Multivariate spatial conditional extremes for extreme ocean environments. *Ocean Eng.* 247:110647

Shooter R, Tawn J, Ross E, Jonathan P. 2021. Basin-wide spatial conditional extremes for severe ocean storms. *Extremes* 24(2):241–65

Simão ML, Sagrilo LVS, Videiro PM. 2022. A multi-dimensional long-term joint probability model for environmental parameters. *Ocean Eng.* 255:111470

Simpson ES, Tawn JA. 2024. Estimating the limiting shape of bivariate scaled sample clouds: with additional benefits of self-consistent inference for existing extremal dependence properties. *Electron. J. Stat.* 18(2):4582–611

Simpson ES, Tawn JA. 2025. Inference for new environmental contours using extreme value analysis. *J. Agric. Biol. Environ. Stat.* 30:638–62

Slater LJ, Anderson B, Buechel M, Dadson S, Han S, et al. 2021. Nonstationary weather and water extremes: a review of methods for their detection, attribution, and management. *Hydrol. Earth Syst. Sci.* 25(7):3897–935

Smit P, Houghton I, Jordanova K, Portwood T, Shapiro E, et al. 2021. Assimilation of significant wave height from distributed ocean wave sensors. *Ocean Model.* 159:101738

Soares CG, Dogliani M, Østergaard C, Parmentier G, Pedersen PT. 1996. Reliability based ship structural design. *Trans. Soc. Naval Archit. Mar. Eng.* 104:357–89

Song Y, Chen J, Sørensen JD, Li J. 2022. Multi-parameter full probabilistic modeling of long-term joint wind-wave actions using multi-source data and applications to fatigue analysis of floating offshore wind turbines. *Ocean Eng.* 247:110676

Song Y, Hong X, Sun T, Zhang Z. 2024. Joint probabilistic modeling of extreme wind-wave conditions under typhoon impact and applications to extreme response analysis of floating offshore wind turbines. *Eng. Struct.* 318:118686

Song Y, Hong X, Xiong J, Shen J, Xu Z. 2023. Probabilistic modeling of long-term joint wind and wave load conditions via generative adversarial network. *Stochast. Environ. Res. Risk Assess.* 37(7):2829–47

Speers M, Randell D, Tawn J, Jonathan P. 2024. Estimating metocean environments associated with extreme structural response to demonstrate the dangers of environmental contour methods. *Ocean Eng.* 311:118754

Tang T, Adcock TA. 2021. Data driven analysis on the extreme wave statistics over an area. *Appl. Ocean Res.* 115:102809

Tang T, Adcock TA. 2022a. A reduced order model for space–time wave statistics using probabilistic decomposition–synthesis method. *Ocean Eng.* 259:111860

Tang T, Adcock TA. 2022b. Estimating space–time wave statistics using a sequential sampling method and Gaussian process regression. *Appl. Ocean Res.* 122:103127

Tendijck S, Eastoe E, Tawn J, Randell D, Jonathan P. 2023. Modeling the extremes of bivariate mixture distributions with application to oceanographic data. *J. Am. Stat. Assoc.* 118(542):1373–84

Tendijck S, Jonathan P, Randell D, Tawn J. 2024a. Temporal evolution of the extreme excursions of multivariate *k*th order Markov processes with application to oceanographic data. *Environmetrics* 35(3):e2834

Tendijck S, Randell D, Feld G, Jonathan P. 2024b. Practical non-stationary extreme value analysis of peaks over threshold using the generalised Pareto distribution: estimating uncertainties in return values. *Ocean Eng.* 312:119247

Towe R, Ross E, Randell D, Jonathan P. 2024. covXtreme: MATLAB software for non-stationary penalised piecewise constant marginal and conditional extreme value models. *Environ. Model. Softw.* 177:106035

Tyralis H, Papacharalampous G. 2024. A review of predictive uncertainty estimation with machine learning. *Artif. Intell. Rev.* 57:94

van Essen SM, Bernardino M, von Bock und Polach F, Campos RM, de Hauteclocque G, et al. 2026. Committee I.1: environment. In *Proceedings of the 22nd International Ship and Offshore Structures Congress*, ed. W Wu, J Ding. Springer

van Essen SM, Monroy C, Shen Z, Helder JA, Kim DH, et al. 2021. Screening wave conditions for the occurrence of green water events on sailing ships. *Ocean Eng.* 234:109218

van Essen SM, Scharnke J, Seyffert HC. 2023a. Required test durations for converged short-term wave and impact extreme value statistics—part 1: ferry dataset. *Mar. Struct.* 90:103410

van Essen SM, Scholcz T, Seyffert HC. 2023b. Prediction of short-term non-linear response using screening combined with multi-fidelity Gaussian process regression. In *Proceedings of the ASME 2023 42nd International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 1: *Offshore Technology*. American Society of Mechanical Engineers van Essen SM, Seyffert HC. 2023. Finding dangerous waves—review of methods to obtain wave impact design loads for marine structures. *J. Offshore Mech. Arct. Eng.* 145(6):060801

Vanem E. 2015. Non-stationary extreme value models to account for trends and shifts in the extreme wave climate due to climate change. *Appl. Ocean Res.* 52:201–11

Vanem E. 2018. A simple approach to account for seasonality in the description of extreme ocean environments. *Mar. Syst. Ocean Technol.* 13(2–4):63–73

Vanem E. 2019. 3-dimensional environmental contours based on a direct sampling method for structural reliability analysis of ships and offshore structures. *Ships Offshore Struct.* 14(1):74–85

Vanem E. 2020. Bivariate regional extreme value analysis for significant wave height and wave period. *Appl. Ocean Res.* 101:102266

Vanem E. 2021. Bivariate regional frequency analysis of sea state conditions. In *Proceedings of the ASME 2021 40th International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 2: *Structures, Safety, and Reliability*. American Society of Mechanical Engineers

Vanem E. 2023a. Analysing multivariate extreme conditions using environmental contours and accounting for serial dependence. *Renew. Energy* 202:470–82

Vanem E. 2023b. Analyzing extreme sea state conditions by time-series simulation accounting for seasonality. *J. Offshore Mech. Arct. Eng.* 145(5):051201

Vanem E, Fazeres-Ferradosa T. 2022. A truncated, translated Weibull distribution for shallow water sea states. *Coast. Eng.* 172:104077

Vanem E, Fekhari E, Dimitrov N, Kelly M, Cousin A, Guiton M. 2023. A joint probability distribution model for multivariate wind and wave conditions. In *Proceedings of the ASME 2023 42nd International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 2: *Structures, Safety, and Reliability*. American Society of Mechanical Engineers

Vanem E, Fekhari E, Dimitrov N, Kelly M, Cousin A, Guiton M. 2024a. A joint probability distribution for multivariate wind-wave conditions and discussions on uncertainties. *J. Offshore Mech. Arct. Eng.* 146(6):061701

Vanem E, Gramstad O, Babanin A, De Bin R, Trulsen K. 2024b. On the distribution of ocean wave crest heights in varying wave conditions. *J. Ocean Eng. Mar. Energy* 10(4):797–815

Vanem E, Lande Ø, Fekhari E. 2024c. A simulation study on the usefulness of the Bernstein copula for statistical modeling of metocean variables. In *Proceedings of the ASME 2024 43rd International Conference on Ocean, Offshore and Arctic Engineering*. Vol. 2: *Structures, Safety, and Reliability*. American Society of Mechanical Engineers

Vanem E, Zhu T, Babanin A. 2022. Statistical modelling of the ocean environment – a review of recent developments in theory and applications. *Mar. Struct.* 86:103297

Wada R, Rohmer J, Krien Y, Jonathan P. 2022. Statistical estimation of spatial wave extremes for tropical cyclones from small data samples: validation of the STM-E approach using long-term synthetic cyclone data for the Caribbean Sea. *Nat. Hazards Earth Syst. Sci.* 22:431–44

Wadsworth JL, Campbell R. 2024. Statistical inference for multivariate extremes via a geometric approach. *J. R. Stat. Soc. Ser. B* 86(5):1243–65

Wang H, Gramstad O, Schär S, Marelli S, Vanem E. 2024. Comparison of probabilistic structural reliability methods for ultimate limit state assessment of wind turbines. *Struct. Saf.* 111:102502

Wang H, Xiao T, Gou H, Pu Q, Bao Y. 2023. Joint distribution of wind speed and direction over complex terrains based on nonparametric copula models. *J. Wind Eng. Ind. Aerodyn.* 241:105509

Wang J, Bai Z, Xie B, Gui J, Gong H, Zhou Y. 2025. Improved inverse first-order reliability method for analyzing long-term response extremes of floating structures. *J. Mar. Sci. Appl.* 24:552–66

Wen Z, Wang F, Wan J, Wang Y, Yang F, Guo C. 2024. Copula-based joint tropical cyclone-induced wind and wave risk analysis: considering the effect of uncertainty using Bayesian inference. *Nat. Hazards* 120:14355–80

Wu Y, Randell D, Christou M, Ewans K, Jonathan P. 2016. On the distribution of wave height in shallow water. *Coast. Eng.* 111:39–49

Yang Z, Dong S. 2023. A novel decomposition-based approach for non-stationary hub-height wind speed modelling. *Energy* 283:129081

Yang Z, Huang W, Dong S, Li H. 2023. Mixture bivariate distribution of wind speed and air density for wind energy assessment. *Energy Convers. Manag.* 276:116540

Zanini E, Eastoe E, Jones MJ, Randell D, Jonathan P. 2020. Flexible covariate representations for extremes. *Environmetrics* 31(5):e2624

Zhang J, Benoit M, Ma Y. 2022. Equilibration process of out-of-equilibrium sea-states induced by strong depth variation: evolution of coastal wave spectrum and representative parameters. *Coast. Eng.* 174:104099

Zhang J, Ma Y, Benoit M. 2024. Statistical distributions of free surface elevation and wave height for out-of-equilibrium sea-states provoked by strong depth variations. *Ocean Eng.* 293:116645

Zhao Y, Dong S. 2021. Design loads and reliability assessment of marine structures considering statistical models of metocean data. *Ocean Eng.* 241:110099

Zhao Y, Dong S. 2022. Comparison of environmental contour and response-based approaches for system reliability analysis of floating structures. *Struct. Saf.* 94:102150

Zhao Y, Dong S. 2023a. Multivariate probability analysis of wind-wave actions on offshore wind turbine via copula-based analysis. *Ocean Eng.* 288:116071

Zhao Y, Dong S. 2023b. Uncertainty analysis of extreme mooring loads associated with environmental contours and peak tension distributions. *Mar. Struct.* 89:103369

Zve ES, Swan C, Hughes GO. 2023. Crest-height statistics in finite water depth. Part 1: the role of the nonlinear interactions in uni-directional seas. *Ocean Eng.* 289:116369