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Spatial dependence of extreme seas in the North East Atlantic from satellite altimeter measurements

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Summary: The extremal spatial dependence of significant wave height in the North East Atlantic is explored using JASON satellite altimeter observations for the period 2002-2018, and a spatial conditional extremes model motivated by the work of Heffernan and Tawn (2004). The analysis involves (a) registering individual satellite passes onto a template transect, (b) marginal extreme value analysis at a set of locations on the template transect and transformation from physical to standard Laplace scale, (c) estimation of the spatial conditional extremes model for a set of locations on a template transect, and (d) comparison of extreme spatial dependence for different template transects. Inferences for two transects considered are qualitatively similar; however, for the 'normal ascending' transect running approximately south-west to north-east lying between Iceland and the UK, extremal spatial dependence is found to decay more quickly than for the second 'opposite descending' transect running approximately north-west to south-east to the west of Ireland.

Keywords: extreme; spatial dependence; significant wave height; satellite; altimeter; Atlantic;

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1. INTRODUCTION

Simultaneous occurrences of extreme ocean events at different locations can involve considerably higher risk than occurrences of an individual extreme event at one location. For example, resources for evacuating coastal regions or de-manning marine facilities can be limited, reducing the efficacy of

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these approaches to effective risk mitigation. There is interest therefore in characterising the extremal spatial dependence of severe seas and related phenomena.

Spatial studies of extreme seas typically rely at least to some extent on hindcast data, since sources of large-scale spatial measurements of extreme seas are not available. Previous studies (e.g. Keresztúri et al. 2016, Ross et al. 2017, Shooter et al. 2019, Shooter et al. 2020b) based on analysis of hindcast data have shown that the nature and extent of extremal dependence in an ocean basin changes with distance between locations, and also potentially with their relative spatial orientation. In recent years, observations of ocean surface roughness from satellite altimetry have become available, providing a more direct source of data for analysis. In particular, observations along transects of the JASON 1, 2 and 3 satellites along approximately the same longitude-latitude path provide a means to estimate extremal spatial dependence not previously available.

Satellite radar altimeters have been shown to be capable of measuring wind speed and significant wave height to high accuracy (Young et al. 2017, Ribal and Young 2019). For Earth observation, the combined data record from multiple satellites now spans more than 30 years and has been shown to provide a high-quality data source for global climatology (Young and Donelan 2018) and extreme value analysis (Takbash et al. 2019, Takbash and Young 2019). A unique characteristic of altimeter remote sensing is that it provides near-instantaneous measurements over spatial domains (along track) of significant wave height. Hence, the data is suited to investigate the spatial variation of wave properties including extreme values.

In this work, we adopt the spatial conditional extremes model of Shooter et al. (2019), Wadsworth and Tawn (2019) and Shooter et al. (2020b), motivated by Heffernan and Tawn (2004), to estimate extremal spatial dependence of significant wave height along satellite transects. This model admits different classes of extremal dependence and is computationally rather straightforward to estimate; it is essentially a non-linear regression model. Other statistical approaches to spatial extremes are motivated by the theory of max-stable processes (MSPs, see e.g. Brown and Resnick 1977, Smith 1990, Schlather 2002, Davison et al. 2012, Ribatet 2013 and Tawn et al. 2018). However, typical MSP models require an assumption that the extremal spatial dependence takes a particular form (known as 'asymptotic dependence', see Section 3); in general, this assumption is unlikely to be appropriate for characterisation of severe ocean environments on large spatial domains. Inverted MSP models are available, but these require the assumption that asymptotic dependence is not present. Other asymptotically dependent spatial extremes models have been proposed (e.g. Reich and Shaby 2012, Ferreira and de Haan 2014, de Fondeville and Davison 2018). Some models in principle are able to describe different classes of extremal dependence (e.g. Wadsworth and Tawn 2012, Wadsworth et al. 2017, Huser and Wadsworth 2019) but in reality are unrealistically restrictive (e.g. in that only one of asymptotic dependence or asymptotic independence can be present at all distances) or computationally unwieldy.

Objectives and outline

The variable of interest in this work is significant wave height (henceforth H_S), a measure of the roughness of the ocean surface at a specific time and location. H_S can be estimated as four times the standard deviation of the ocean surface over an interval of observation of the order of one hour. H_S varies slowly in space (over tens of kilometres) and in time (over hours). The objective of the current work is to apply the spatial conditional extremes model to quantify the extremal spatial dependence of significant wave height in the North East Atlantic as measured by satellite altimetry.

The layout of the paper is as follows. Section 2 introduces the JASON altimeter data, and describes the pre-processing of data necessary in advance of extreme value modelling. The spatial conditional extremes model is described in Section 3. Results of applying the spatial conditional extremes model for two North East Atlantic transects are then given in Section 4. Section 5 provides discussion and conclusions. Software, data and supporting results are provided in a GitHub repository (Shooter et al. 2020a).

2. DATA

2.1. JASON satellite altimeter missions

JASON (Joint Altimetry Satellite Oceanography Network) 1, 2 and 3 altimeter measurements of the ocean's surface are used in this work. JASON-1 was the successor altimeter mission to TOPEX/Poseidon, which measured ocean surface topography from 1992 to 2005. Like its predecessor, JASON-1 was a joint project between NASA and the French space agency CNES. JASON-1 was launched in December 2001 and decommissioned in June 2013. In the initial calibration phase of follow-up mission JASON-2 (launched in June 2008), both JASON-1 and JASON-2 satellites were placed in the same track and phased approximately 56 seconds apart. However, the JASON-1 orbit started to drift in January 2009. After approximately 11 years of operation, JASON-2 was decommissioned in October 2019. To ensure continuity of high-quality measurements for ocean science applications, JASON-3 was launched in January 2016 and is still operational. All three JASON missions were placed in near-polar orbits, covering the globe from latitude 66°S to 66°N. The exact repeated mission, inclination and altitude for all satellites were 10 days, 66° and 1336 km. The altimeters on all three JASON satellites were similar, with dual systems using separate antennas operating in the Ku-band (13.575 GHz) and C-band (5.3 GHz).

All H_S data measured by JASON-1, JASON-2 and JASON-3 were sourced from the Australian Ocean Data Network (AODN). These datasets have been calibrated and quality controlled as described by Ribal and Young (2019). The database is updated every six months and presently includes data up to the end of June 2020. Each along-track measurement is allocated a data quality flag designating an observation as one of 'good', 'probably good', 'bad' and 'missing'. The present analysis uses 'good' data only. The analysis of Ribal and Young (2019) calibrated altimeter measurements against U.S. National Data Buoy Centre (NDBC) buoy data, applying a linear regression correction to the raw H_S data from the missions. In addition, the validation analysis conducted by Ribal and Young (2019) investigated the performance of the altimeters at extreme H_S up to 9m. The analysis showed no indication of degradation of the signals at these extreme values.

The altimeter tracks from JASON-1, JASON-2 and JASON-3 used in the current study were selected from an area bounded by longitudes from 330°E to 360°E, and latitudes from 45°N to 65°N. Two specific transects were used in the analysis. The 'normal' transect (henceforth referred to as 'SWNE') is an ascending south-west to north-east track whereas the 'opposite' transect (henceforth 'NWSE') descends from north-west to south-east. Due to orbital drift, the tracks for JASON-1 and JASON-2 vary spatially whereas the track of JASON-3 is more stable. The duration of the available data from each satellite is summarized in Table 1, together with the numbers of transects available for analysis.

[Table 1 about here.]

2.2. Transect registration and marginal transformation

We estimate extremal spatial dependence for a small set of p 'registration locations' $r_R(j)$, $j = 0, 1, ..., q$ on a template transect, where $p = q + 1$. We define registration locations as shown in Figure 1. These are approximately equidistant between adjacent JASON-3 transects and approximately equally spaced in the transect direction, for the SWNE and NWSE tracks. For each available satellite pass $r_\ell(t)$, $t \in$ time period T_ℓ, and pass index $\ell = 1, 2, ...$ over the region of interest, we find the nearest point $r_\ell(t_j^*)$ on the satellite transect to each of the p registration locations $r_R(j)$, $j = 0, 1, ..., q$ such that

$$t_j^* = \underset{t \in T_\ell}{\text{argmin}} \ \text{dist}(r_\ell(t), r_R(j)) \quad (1)$$

with physical distance dist calculated using the spherical law of cosines, and allocate the value of H_S measured at $r_\ell(t_j^*)$ to $r_R(j)$. For any satellite pass ℓ, if the maximum of $r_{\ell j}$ exceeds 50 km the pass is not registered. In this way, we obtain 'registered transects' for each pass of JASON 1, 2 and 3 over the region of interest, and a sequence of p measurements of H_S at the registration locations for each transect. The total number of transects included is 691, 581 and 125 for each of JASON 1, 2 and 3 for the SWNE track, and 588, 559 and 127 for the NWSE track. Figure 1 shows $p = 19$ registration locations for each transect, the first of which (location 0) is referred to as the 'reference location' or 'conditioning location'.

[Figure 1 about here.]

The prevailing wind and storm direction in this mid-latitude region is from the south-west or west. The SWNE transect therefore is typically more aligned with the direction of storm propagation, whereas the NWSE transect lies approximately perpendicular to it, and therefore approximately parallel to the region of highest H_S (e.g. Young 2017). H_S on the SWNE transect is also more likely to be influenced by the presence of the UK and Icelandic land masses.

Scatter plots of H_S from subsets of locations on the SWNE and NWSE tracks are shown in Figures 2 and 3.

[Figure 2 about here.]

[Figure 3 about here.]

We see that the dependence between H_S at different locations decreases with distance between locations. However there is a positive association between H_S values even when locations are separated by more than 1000 km. It appears that the strength of dependence between locations with given separation does not vary obviously along the transect.

Marginal extreme value analysis was performed independently for sample data for each of the $2p$ registration locations, and samples thereby transformed to marginal Laplace scale with $\dot{X}$ representing H_S at a location on physical scale. A generalised Pareto (GP) distribution with shape ξ, scale σ and high threshold ψ was fitted to the sample at each registration location. The GP (conditional) cumulative distribution function is $F_{\dot{X}|\dot{X}>\psi}(\dot{x};\xi,\sigma,\psi) = 1-(1+(\xi/\sigma)(\dot{x}-\psi))_+^{-1/\xi}$ for $\dot{x}>\psi$, $\psi\in(-\infty,\infty)$, $\xi\in(-\infty,\infty)\setminus\{0\}$, $\sigma\in(0,\infty)$ and $(y)_+ = y$ for $y>0$ and $=0$ otherwise. When $\xi=0$, the conditional distribution takes the form $1-\exp[-(x-\psi)/\sigma)]$. The sensitivity of estimated ξ to choice of threshold ψ was found to be small for ψ corresponding to the GP quantiles with non-exceedance probabilities τ around 0.7. Marginal analysis suggests that there is some spatial non-stationarity in H_S. To illustrate this, Figure 4 gives 100-year return value estimates (as bootstrap median and 95% uncertainty bands) for the SWNE and NWSE tracks. The T-year return value is the quantile of the annual distribution of a quantity (H_S here) with non-exceedance probability $1-1/T$.

[Figure 4 about here.]

It is interesting that the maximum value of H_S ever reported by buoy measurement is approximately 19m, recorded at the location of UK Met Office buoy K5 at approximately 59°N, 12°W in 2013 (WMO n.d.), near to the start of the SWNE track. The unconditional cumulative distribution function $F_{\dot{X}}$ of H_S at a location is then given by $F_{\dot{X}}(\dot{x}) = \tilde{F}_{\dot{X}}(\dot{x})$ if $\dot{x}\le\psi$ and $(1-\tau)F_{\dot{X}|\dot{X}>\psi}(\dot{x})$ otherwise, where $\tilde{F}_{\dot{X}}$ is an empirical ('counting') estimate for the cumulative distribution of threshold non-exceedances. Marginal transformation to Laplace-scale variate X is achieved at each registration location using $F_X(x) = F_{\dot{X}}(\dot{x})$ for $x,\dot{x}\in\mathbb{R}$, where F_X is the cumulative distribution function of the standard Laplace distribution, given by $F_X(x) = (1/2)\exp(x)$ if $x\le 0$ and $1-(1/2)\exp(-x)$ otherwise.

Figures 5 and 6 show scatter plots of threshold exceedances of Laplace-scale H_S at a selection of remote locations on H_S at the reference location for each of the SWNE and NWSE tracks. The threshold level used at the reference location corresponds to the quantile of the Laplace distribution with non-exceedance probability 0.7. Both figures show that dependence between Laplace-scale H_S reduces with increasing distance. For small distances, the dependence is high with data lying approximately on the line $y=x$. At the very largest distances, there is also evidence for a small level of positive dependence. We would expect the estimated spatial conditional extremes model to reflect these features.

[Figure 5 about here.]

[Figure 6 about here.]

3. MODEL AND INFERENCE

3.1. Extremal spatial dependence

Severe wind-sea conditions in the North East Atlantic are generated by relatively large-scale atmospheric low pressure systems. Hence for two nearby locations, an extreme value of H_S observed at one location might be expected to be associated with an extreme observation at another. Extremes observed at distant pairs of locations are less likely to be related. We can calculate the probability that given an observation of an extreme event at one location, we observe extreme events simultaneously at other locations, providing a measure of dependence between the events. Coles et al. (1999) introduce the measures χ and $\overline{\chi}$, estimated via their sub-asymptotic forms $\chi(u)$ and $\overline{\chi}(u)$, where $u \in [0, 1]$, to describe extremal dependence. For bivariate uniform random variables (U, V) these are defined as $\chi(u) = 2 - \log \mathbb{P}(U < u, V < v)/\log \mathbb{P}(U < u)$ and $\overline{\chi}(u) = 2 \log \mathbb{P}(U > u)/\log \mathbb{P}(U > u, V > v) - 1$. χ and $\overline{\chi}$ may be obtained by taking the respective limits of these functions, as $u \to 1$. The nature of extremal dependence between U and V may then be classified by considering χ and $\overline{\chi}$ together. If $\chi = 0$ and $-1 \leqslant \overline{\chi} < 1$, the random variables are asymptotically independent (AI), and the value of $\overline{\chi}$ signifies the level of dependence. On the other hand, if $\overline{\chi} = 1$ and $0 < \chi \leqslant 1$, then the pair (U, V) exhibit asymptotic dependence (AD), with χ providing a measure of this. See Ledford and Tawn (1996), Coles et al. (1999) and Tawn et al. (2018) for further details.

3.2. Multivariate conditional extremes

Suppose now we have a vector of random variables $(X_0, \mathbf{X})$, where X_0 and $\mathbf{X} = (X_1, \ldots, X_q)$ temporarily have Gumbel marginal distributions, and random variables $\mathbf{Z} = (\mathbf{X} - \mathbf{a}(X_0))/\mathbf{b}(X_0)$ for functions $\mathbf{a} : \mathbb{R} \to \mathbb{R}^q$ and $\mathbf{b} : \mathbb{R} \to \mathbb{R}^q$, where all operations are taken to be component-wise. Heffernan and Tawn (2004) assume the existence of $\mathbf{a}$ and $\mathbf{b}$ such that, for $x > 0$, $\lim_{u\to\infty} \mathbb{P}(\mathbf{Z} \leqslant \mathbf{z}, X_0 - u > x|X_0 > u) = G(\mathbf{z}) \exp(-x)$, where G is a joint distribution with non-degenerate margins; asymptotic justification is given by Heffernan and Tawn (2004) and Heffernan and Resnick (2007). Keef et al. (2013) show that if $(X_0, \mathbf{X})$ has Laplace-distributed margins then canonical functional forms for $\mathbf{a}(\cdot)$ and $\mathbf{b}(\cdot)$ are $\mathbf{a}(x) = \boldsymbol{\alpha} x$ and $\mathbf{b}(x) = x^{\boldsymbol{\beta}}$ (for $x > 0$), where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_q)$ and

$\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_q)$, and $\alpha_j \in [-1, 1]$ and $\beta_j \in (-\infty, 1]$. We assume positive dependence, restricting $\alpha_j \in [0, 1], \beta_j \in [0, 1]$ for all 'remote locations' $j \in \{1, 2, \ldots, q\}$. For some high threshold u and all $x_0 > u$, we assume the model form $\mathbf{X}|\{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\boldsymbol{\beta}} \mathbf{Z}$, where $\mathbf{Z} \sim G$ is independent of X_0. Different values for (α_j, β_j) indicate different classes of extremal dependence as follows: $(\alpha_j, \beta_j) = (1, 0)$ corresponds to AD, $\alpha_j = 0$ to perfect independence, and intermediate values of α_j to AI. Thus, on Laplace scale, for positive dependence: (a) AD corresponds to X_j and X_0 for large X_0 growing at the same rate with conditional extremes slope parameter $\alpha_j = 1$, $\beta_j = 0$, (b) AI corresponds to X_j growing more slowly than X_0, with $\alpha_j \in (0, 1)$, and (c) perfect independence corresponds to X_j not growing with X_0, and $\alpha_j = 0$.

3.3. Spatial conditional extremes

We extend the multivariate conditional extremes model outlined in Section 3.2 to a spatial context using the spatial conditional extremes (SCE) model described by Tawn et al. (2018), Wadsworth and Tawn (2019) and Shooter et al. (2020b). Specifically, we represent H_S on the SWNE and NWSE transects by spatial process $X(\cdot)$ on domain $\mathcal{S}$ with Laplace marginal distributions. Then for distance $d \in \mathbb{R}_{\geq 0}$ between locations $r, r' \in \mathcal{S}$, with positive dependence between variables, for all $x_0 > u$, $X(r') \mid \{X(r) = x_0\} = \alpha(d) x_0 + x_0^{\beta(d)} Z(r' - r)$, where $\alpha : \mathbb{R}_{>0} \to [0, 1]$, $\beta : \mathbb{R}_{>0} \to [0, 1]$ and $Z(\cdot)$ is a residual process independent of $X(\cdot)$. To make inferences using vector $\mathbf{X}$ of random variables observed at registration locations $\{r_R(j)\}_{j=0}^q$, $X(\cdot)$ is treated as finite-dimensional. We set $d_j = \text{dist}(r_R(j), r_R(0))$ for $j = 1, 2, \ldots, q$ for distance $\text{dist}(\cdot, \cdot)$ between locations, $\alpha_j = \alpha(d_j)$ and $\beta_j = \beta(d_j)$. We assume that $Z(\cdot)$ has delta-Laplace (or generalised Gaussian) margins with parameters μ, σ and δ dependent on d, which need to be estimated. The marginal density f_{Z_j} of Z at distance d_j is

$$f_{Z_j}(z_j) = \frac{\delta_j}{2\kappa_j \sigma_j \Gamma\left(\frac{1}{\delta_j}\right)} \exp\left\{ -\left| \frac{z - \mu_j}{\kappa_j \sigma_j} \right|^{\delta_j} \right\} \tag{2}$$

for $j = 1, 2, \ldots, q$, $\delta_j, \sigma_j, \kappa_j \in \mathbb{R}_{>0}, \mu_j \in \mathbb{R}$, $z_j \in \mathbb{R}$, where $\kappa_j^2 = \Gamma(1/\delta_j) / \Gamma(3/\delta_j)$ and $\Gamma(\cdot)$ represents the gamma function. The mean and variance of this distribution are respectively μ_j and σ_j^2, regardless of the choice of δ_j, and the distribution is henceforth denoted by $\text{DL}(\mu_j, \sigma_j^2, \delta_j)$. The case $\delta_j = 2$ corresponds to a Gaussian distribution, and $\delta_j = 1$ to a Laplace distribution; the standard Laplace distribution with variance 2 corresponds to $\sigma_j^2 = 2$ in our notation. As $d \to \infty$, we approach perfect independence between locations with $\mu_j \to 0$, and $\delta_j \to 1$, $\sigma_j \to \sqrt{2}$ (i.e. standard Laplace), and also $\alpha_j, \beta_j \to 0$. The model is not informative for δ_j at $d = 0$.

Now consider the vector $\mathbf{X}$ corresponding to $p = q + 1$ registration locations with standard Laplace marginal distributions $X_j \sim \text{DL}(0, 2, 1)$ for $j = 0, \ldots, q$. We assume, conditional on $X_0 = x_0$, for $x_0 > u$, that $(X_1, \ldots, X_q)|\{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\boldsymbol{\beta}} \mathbf{Z}$ where $\mathbf{Z} \sim \text{DL}_q(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{\delta}; \boldsymbol{\Sigma})$. $\boldsymbol{\Sigma}$ is the $q \times q$ correlation matrix for a Gaussian dependence structure between residual components, and

$$F_{\mathbf{Z}}(\mathbf{z}) = \Phi_q \left(\Phi^{-1}(F_{Z_1}(z_1)), \Phi^{-1}(F_{Z_2}(z_2)), \ldots, \Phi^{-1}(F_{Z_q}(z_q)); \mathbf{0}, \boldsymbol{\Sigma} \right) \tag{3}$$

where F represents a cumulative distribution function and Φ is the cumulative distribution function of a standard Gaussian distribution. $\Phi_q(\mathbf{0}, \boldsymbol{\Sigma})$ is the cumulative distribution function of a q-dimensional Gaussian distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$. The j, j' element $\Sigma_{jj'}$ of residual correlation matrix $\boldsymbol{\Sigma}$, $j, j' = 1, 2, \ldots, q$ quantifies the dependence between SCE residuals (on standard Gaussian-scale) at registration locations $r_R(j)$ and $r_R(j')$ given conditioning on location $r_R(0)$. Based on the findings of Shooter et al. (2020b), we expect that the value of $\Sigma_{jj'}$ depends on both the distance $\text{dist}(r_R(j), r_R(j'))$ between remote locations and distances $\text{dist}(r_R(j), r_R(0))$, $\text{dist}(r_R(j'), r_R(0))$ from remote locations to conditioning site. When $\text{dist}(r_R(j), r_R(0))$ and $\text{dist}(r_R(j'), r_R(0))$ are large relative to $\text{dist}(r_R(j), r_R(j'))$, the conditional correlation between remote locations will be similar to the unconditional correlation. Otherwise we expect conditioning of $r_R(0)$ to influence correlation between process at $r_R(j)$ and $r_R(j')$. We therefore adopt a parameterisation for $\boldsymbol{\Sigma}$ equivalent to the correlation function for a standard Gaussian field evaluated at $p = q + 1$ locations conditioned on its value at one location, with powered exponential dependence. With $\boldsymbol{\Sigma}^*$ representing the $p \times p$ correlation matrix of the unconditioned field (and matrix indexing starting from zero for convenience), the correlation matrix $\boldsymbol{\Sigma}$ for the conditional field has elements given by $\Sigma_{jj'} = (\Sigma^*_{jj'} - \Sigma^*_{j0}\Sigma^*_{j'0})(1 - \Sigma^{*2}_{j0})^{-1/2}(1 - \Sigma^{*2}_{j'0})^{-1/2}$ for $j, j' = 1, 2, \ldots, q$ with conditioning location indexed by zero. Further, we assume that the correlation between observations at different locations in the unconstrained field reduces as a function of the distance between locations, with powered exponential form $\Sigma^*_{jj'} = \exp[-(\text{dist}(r_R(j), r_R(j'))/\rho_1)^{\rho_2}]$ for $\rho_1, \rho_2 \in \mathbb{R}_{>0}$ and $j, j' = 0, 1, 2, \ldots, q$, for parameters ρ_1 and ρ_2 to be estimated.

Marginally, writing X_j^c to represent $X_j|\{X_0 = x_0\}$, $j = 1, 2, \ldots, q$, we have $X_j^c = \alpha_j x_0 + x_0^{\beta_j} Z_j \sim$

$\text{DL}(m_j, s_j^2, \delta_j)$, where $Z_j \sim \text{DL}(\mu_j, \sigma_j^2, \delta_j)$ for $j = 1, 2, ..., q$, so that $Z_j = Z_j^* \sigma_j + \mu_j$, where $Z_j^* \sim \text{DL}(0, 1, \delta_j)$. Conditional means m_j and standard deviations s_j are given by

$$m_j = \alpha_j x_0 + x_0^{\beta_j} \mu_j \quad \text{and} \quad s_j = x_0^{\beta_j} \sigma_j. \tag{4}$$

Hence

$$\mathbf{X}^c = (\mathbf{X} | \{X_0 = x_0\}) \sim \text{DL}_p(\mathbf{m}, \mathbf{s}^2, \boldsymbol{\delta}; \boldsymbol{\Sigma}) \tag{5}$$

for $x_0 > u$, a q-dimensional delta-Laplace distribution with mean $\mathbf{m} = \{m_j\}_{j=1}^q$, variance $\mathbf{s}^2 = \{s_j^2\}_{j=1}^q$, delta parameters $\boldsymbol{\delta} = \{\delta_j\}_{j=1}^q$ and (standard Gaussian-scale) covariance $\boldsymbol{\Sigma}$.

Given a large value of a random variable at the conditioning location, the spatial conditional extremes model thus describes the dependence between values across the full set of locations in two ways. The relationship between the value at each individual remote location and that at the conditioning location is characterised as a function of the distance between the remote and conditioning locations, using the conditional extremes model. The dependence between values at pairs of remote locations is also characterised (using a Gaussian process after variable transformation) as a function of the distance between those remote locations.

3.4. Inference

We use Bayesian inference to estimate the joint posterior distribution of the spatial conditional extremes model parameters $\Omega = \{\{\alpha_j, \beta_j, \mu_j, \sigma_j, \delta_j\}_{j=1}^q, \rho_1, \rho_2\}$. To achieve this, we need to derive the sample likelihood for the SCE model, and make a reasonable prior specification for Ω. The description in Section 3.3 is sufficient to evaluate the likelihood for a sample of observations from the registration locations on the SWNE and NWSE transects. As detailed in Shooter et al. (2020b), we differentiate Equation 3 to find the joint density function of residuals, and hence using Equation 5 we find the density $f_{\mathbf{X}^c}(\mathbf{x})$ for any observation $\mathbf{x}$ over the p locations. For (Laplace-scale) sample $\{x_i j\}_{i=1,j=0}^{n,p}$, the negative log-likelihood is

$$\frac{np}{2}\log(2\pi) + \frac{n}{2}\log|\boldsymbol{\Sigma}| + \sum_{j=1}^{p}\log\left(2s_j\kappa\Gamma\left[\frac{1}{\delta_j}\right]\right) - n\sum_{j=1}^{p}\log\delta_j$$
$$+\sum_{i=1}^{n}\left\{\mathbf{w}_i'\boldsymbol{\Sigma}^{-1}\mathbf{w}_i + \sum_{j=1}^{p}\left|\frac{x_{ij}-m_j}{\kappa_j s_j}\right|^{\delta_j} + \sum_{j=1}^{p}\log\phi(w_{ij})\right\} \quad (6)$$

where $\kappa_j^2 = \Gamma(1/\delta_j)/\Gamma(3/\delta_j)$ for each j and $\mathbf{w}_i' = (w_{i1}, w_{i2}, \ldots, w_{ip})$, where $w_{ij} = \Phi^{-1}\{F_{X_j^c}(x_{ij})\}$.

The variation of each of α, β, μ, σ and δ with distance is characterised using a piecewise linear form consisting of $n_d = 10$ equally spaced nodes over appropriate physical distance domains for the SWNE and NSWE transects. Sensitivity of inference to the choice of n_d near 10 was explored and found to be small. A total of $n_d \times 5 + 2$ parameters need to be estimated.

We optionally also restrict the space of feasible combinations $\{\alpha_j, \beta_j\}$, $j = 1, 2, ..., q$ for each conditioning location to ensure that conditional quantiles from AI models do not exceed those from AD models, as proposed by Keef et al. (2013) (see e.g. Shooter et al. 2019 for further details in a spatial context).

An adaptive MCMC algorithm is used for parameter inference. Briefly, random search is used to find a reasonable starting solution. Then a Metropolis-within-Gibbs algorithm is used iteratively to sample each of the elements of Ω in turn for a total of $n_{MiG} = 250$ iterations. Subsequently we use the adaptive Metropolis scheme of Roberts and Rosenthal (2009) to jointly update the full set Ω of parameters for a further $n_{GA} = 50,000$ iterations. Uniform prior distributions on plausible domains are used for model parameters. Chain convergence is judged to have occurred when trace plots for parameters and their dependence stabilise. Fuller description of the MCMC scheme is given in Shooter et al. (2020b). MATLAB software, altimeter data and illustrative results are available at Shooter et al. (2020a).

4. RESULTS

Parameter estimates for the spatial conditional extremes model estimated for the SWNE and NWSE samples are shown in Figure 7 using a threshold u corresponding to non-exceedance probability 0.7, with the conditional quantile constraints of Keef et al. (2013) imposed. We expect from simple physical considerations, at small distances d, that the characteristics of H_S at the remote location are similar to those at the reference location; in terms of model parameters, this implies $\alpha \approx 1$ and $\beta = \mu = \sigma \approx 0$. At large d, we expect that data from the two locations would correspond to observations from independent standard Laplace distributions, with $\alpha = \mu \approx 0$, $\beta \approx 0$, $\sigma \approx \sqrt{2}$ and $\delta \approx 1$. The parameter estimates found appear to be generally consistent with these expectations. The decay of estimated α with d is greater for the SWNE transect, but this is compensated for by larger estimates for μ at intermediate d. Estimates of β reduce from approximately 0.3 and 0.2 for SWNE and NWSE, to zero with increasing d. Estimates of δ decay from approximately 1.8 and 1.4 to 1.0. The estimated value of σ increases with d from approximately 0.2 to $\sqrt{2}$ for both SWNE and NWSE, but the latter reaches the asymptote sooner after about 700 km. Estimates for parameters ρ_1 and ρ_2 of the conditional Gaussian residual process for SWNE and NWSE are similar.

[Figure 7 about here.]

[Figure 8 about here.]

A number of sensitivity studies were conducted to examine the stability of inferences to different sources of variation; these are reported in Shooter et al. (2020a). In particular, other threshold choices yield similar parameter estimates and trends with distance. Further, the effect of removing the conditional quantile constraints of Keef et al. (2013) was examined and found to be small, as also noted previously in Shooter et al. (2020b). Finally, the effect of changing the conditioning location was examined: general trends observed were consistent with those in Figure 7. To illustrate this specifically, Figure 8 gives estimates for parameters α (top) and σ (bottom) resulting from choosing conditioning locations indexed by 0 (west-most location on transect, see Figure 1), 9 (central location) and 18 (east-most location). Comparing across columns in Figure 8 we see that, regardless of conditioning location, the rate of decay of α with distance for the SWNE transects (black) is greater than for NWSE transects (orange); the same is true for the rate of increase of σ with distance. Note that for conditioning location 9, due its position half way along a transect, we have a smaller range of distances for model fitting, resulting in high parameter uncertainty for large distances (equivalent to the specified prior distribution). Plots for the remaining parameters with distance (see Shooter et al. 2020a) do not show such obvious differences between SWNE and NWSE transects.

There is a strong suggestion from Figure 7 that, on the SWNE transect, a remote location at a distance > 750 km from the reference location is effectively independent of it since the estimate of

α, β and μ are near zero, the estimate of δ near 1 and the estimate of σ near $\sqrt{2}$. On the NWSE transect, the distance between locations needs to be > 1500 km before these conditions are satisfied.

Figure 9 shows the corresponding Laplace-scale conditional mean and standard deviation as a function of distance (see Equation 4) for the analysis, for conditioning value x_0 with non-exceedance probability 0.9. In terms of conditional mean, the difference between the SWNE and NWSE transect estimates reflects the difference in estimated α parameters. For conditioning quantile with non-exceedance probability 0.7 (not shown), there is little difference between the two conditional mean profiles; in this case, the decay of the SWNE conditional mean with distance takes more of an 'S' shape, with slower initial decay and more rapid decay at intermediate distances; the conditional mean decay for the NWSE transect is almost linear. Differences in conditional standard deviations with distance for conditioning quantile with non-exceedance probability 0.9 (and 0.7) reflect the differences in σ observed in Figure 7.

[Figure 9 about here.]

Figure 10 shows Laplace-scale observations and corresponding trajectories simulated under the fitted models for the SWNE (left) and NWSE transects in terms of 5 illustrative quantile levels, for conditioning values x_0 corresponding to non-exceedance probabilities $\in (0.7, 0.75]$. There is good agreement between observation and simulation in general.

[Figure 10 about here.]

Figure 11 shows the conditional mean and standard deviation of extreme H_S for SWNE and NWSE transects on the physical scale, assuming that $H_S = 10$m is observed at the reference location. This is achieved by transforming the corresponding Laplace-scale profiles to physical scale using the estimated generalised Pareto marginal models at each registration location in turn. A value of $H_S = 10$m corresponds to a non-exceedance probability of approximately 0.98 at the reference location on both transects. There is strong evidence that the conditional mean on the SWNE transect decays more quickly than on the NWSE transect, but that the conditional standard deviations are very similar with distance.

[Figure 11 about here.]

Takbash et al. (2019) estimated a generalised Pareto distribution for peaks-over-threshold from a 30-year altimeter record to investigate global values of 100-year return value of H_S. The present results (Figure 4) are consistent in magnitude with estimates from Takbash et al. (2019), yielding slightly higher values in the region of the NWSE compared to the SWNE transect. The NWSE transect is in a relatively more exposed region of the open North Atlantic; Takbash et al. (2019) show that values of 100-year return value for H_S in this region are more uniform along-track than for the SWNE transect. For the SWNE transect, the proximity of Icelandic and UK land masses result in less severe extreme conditions. This may also account for the longer spatial scale (1500 km) determined in the present analysis for the NWSE transect compared to SWNE (750 km).

5. DISCUSSION AND CONCLUSIONS

In this work we use a spatial conditional extremes model to quantify spatial dependence of extreme values of altimeter measurements of H_S in the North East Atlantic. Combining observations from JASON 1, 2 and 3 for two template transects each consisting of 19 registration locations, we find that model parameters α, β, μ, σ and δ all vary slowly with distance from a reference location (taken to be the most westerly point on each transect). Parameter estimates for each transect suggest that neighbouring locations exhibit near-asymptotic dependence, transitioning to asymptotic independence and eventually full independence with increasing distance. We find strong evidence for differences in extremal spatial characteristics along the transects. For one transect (SWNE) passing from the south-west between the UK and Iceland, the conditional mean profile given a large value at the south-western reference location decays relatively rapidly with distance; specifically, from 10m to approximately 5m over 750 km. For a second transect (NWSE) passing the west coast of Ireland from the north-west, the corresponding decay is more gradual; from 10m to approximately 5m over 1700 km.

The conditional extremes model requires transformation of observations to standard marginal scale prior to analysis. This is achieved by fitting generalised Pareto models using maximum likelihood estimation at each registration location independently. The estimated marginal models indicate that 100-year return values for H_S exceeding 20m are to be expected, particularly on the NWSE transect. The conditional extremes model then uses asymptotic arguments to motivate a functional form for the variation of one or more random variables given extreme values of a different conditioning random variable on standard marginal scale. A conditional Gaussian process provides a good representation for residual dependence.

We find that inferences are relatively stable to choice of exceedance threshold for the conditional extremes model, for thresholds with non-exceedance probabilities of at least 0.7. Further we find that imposition of conditional quantile constraints do not materially influence fits. Software, data and illustrations of related sensitivity studies are given at Shooter et al. (2020a) so that readers can explore the inference further if interested.

The choice of conditioning location for a spatial conditional extremes analysis is in general arbitrary. In this study, a conditioning location was selected at one end of each transect to (a) create a balanced sample for analysis with approximately equal numbers of pairs of observations at any distance, and (b) maximise the largest distance available in the sample so that a model for the far field could be estimated as well as possible. We note that, as in Shooter et al. (2019), a pooled analysis could be performed in which a pseudo-likelihood over all possible conditioning locations is adopted. Pooling in this manner complicates the quantification of uncertainty and was not considered here, but is a useful approach e.g. when sample size is small.

Compared with gridded hindcast data, an altimeter samples the ocean surface at a low rate of approximately 10 days relative to the time interval (1-3 days) corresponding to a typical north Atlantic storm. The spatial resolution of altimeter measurements along-track is approximately 10 km. However, the cross-track resolution is low, at approximately 500 km. As a result, some storm events may be under-sampled or missed completely as discussed by Young et al. (2017), Young et al. (2011), Vinoth and Young (2011) and Young et al. (2012). This is a limitation for complete examination of extremes, but may not be problematic in estimation of joint spatial tails of H_S if under-sampling is assumed to occur at random. The quality of altimeter data has been examined in numerous studies, and shown by Ribal and Young (2020) to exhibit smaller random errors than either scatterometer or NDBC buoy data for wind fields. Given these limitations, the process of estimating extreme value models and return values directly from altimeter data is straightforward. Relative to extreme value analysis of hindcast data, the most notable extra task in the current application was to map JASON transects onto the registration locations on specified SWNE and NWSE template transects of interest, itself the source of some extra variation.

No attempt was made in the current work to account for the effects of covariates (like storm direction and season), which have been found to be influential in marginal extreme value inference (e.g. Feld et al. 2015); a combination of altimeter and scatterometer data sources (e.g. Ribal and Young 2020) may be exploited to this end. There is also the opportunity to combine altimeter and hindcast data for inference purposes, and to exploit the conditional extremes model of Heffernan and Tawn (2004) directly for joint modelling and calibration of H_S values from multiple sources including in-situ measurements. Jones et al. (2018) illustrates how different sources might be combined for thorough uncertainty quantification.

When there is interest in estimating the joint spatial environmental risk (e.g. along a coastline, or to multiple marine installations, e.g. Kereszturi et al. 2016) from an extreme storm, the spatial conditional extremes model in this work provides a relatively straightforward approach built on established asymptotic results. As illustrated here, altimeter measurements provide a useful high-quality resource for the examination of the spatial structure of wave fields.

6. ACKNOWLEDGEMENTS

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FIGURES

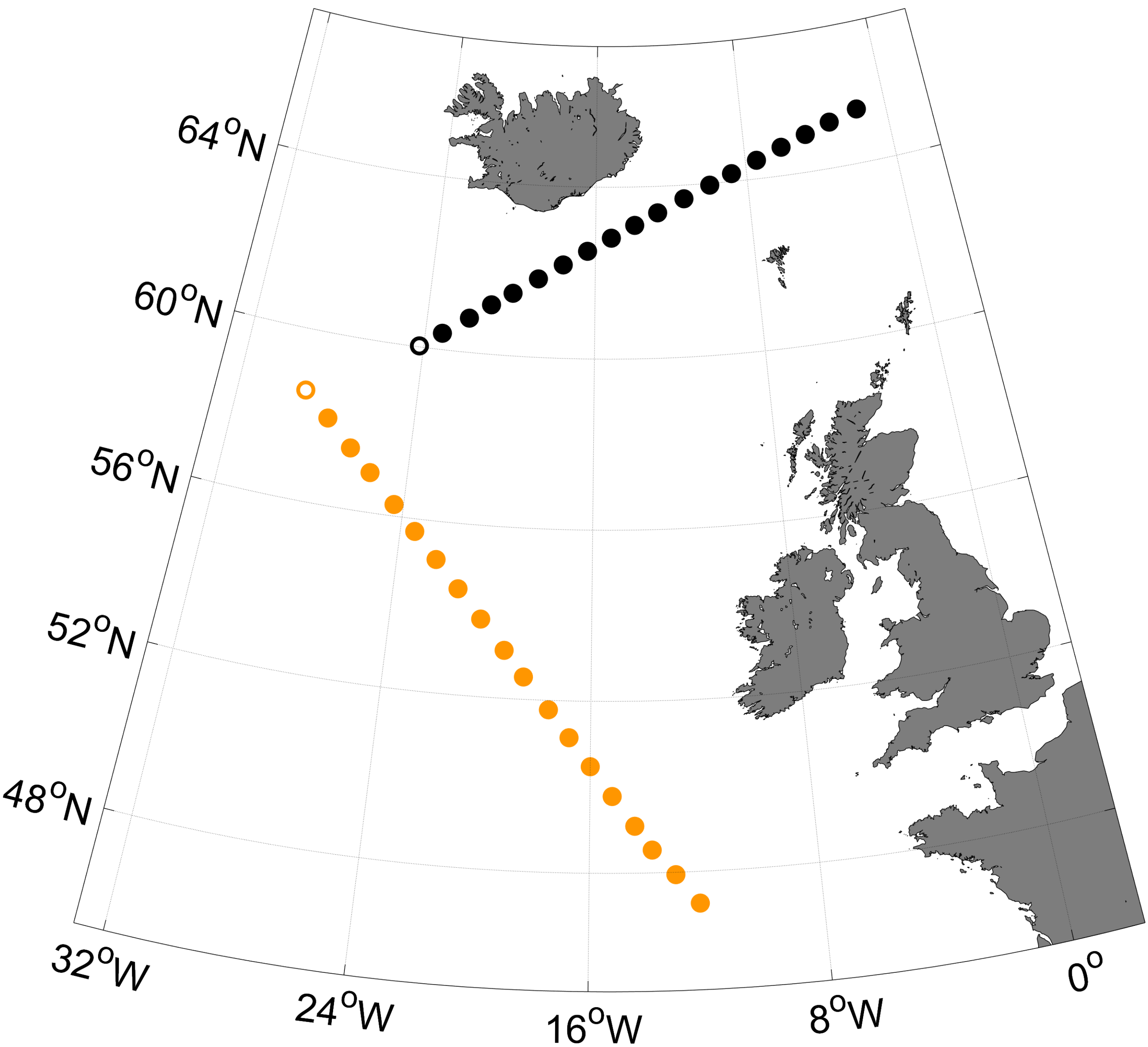

Figure 1. Locations of registered satellite measurements on two template transects. Normal ascending SWNE (black) and opposite descending NWSE (orange) transects shown. 'Registration locations' are numbered 0,1,2,...,18 for each transect. The first registration location, referred to as the 'reference location', 'conditioning location', or 'location 0' for the transect, is shown as a hollow disc.

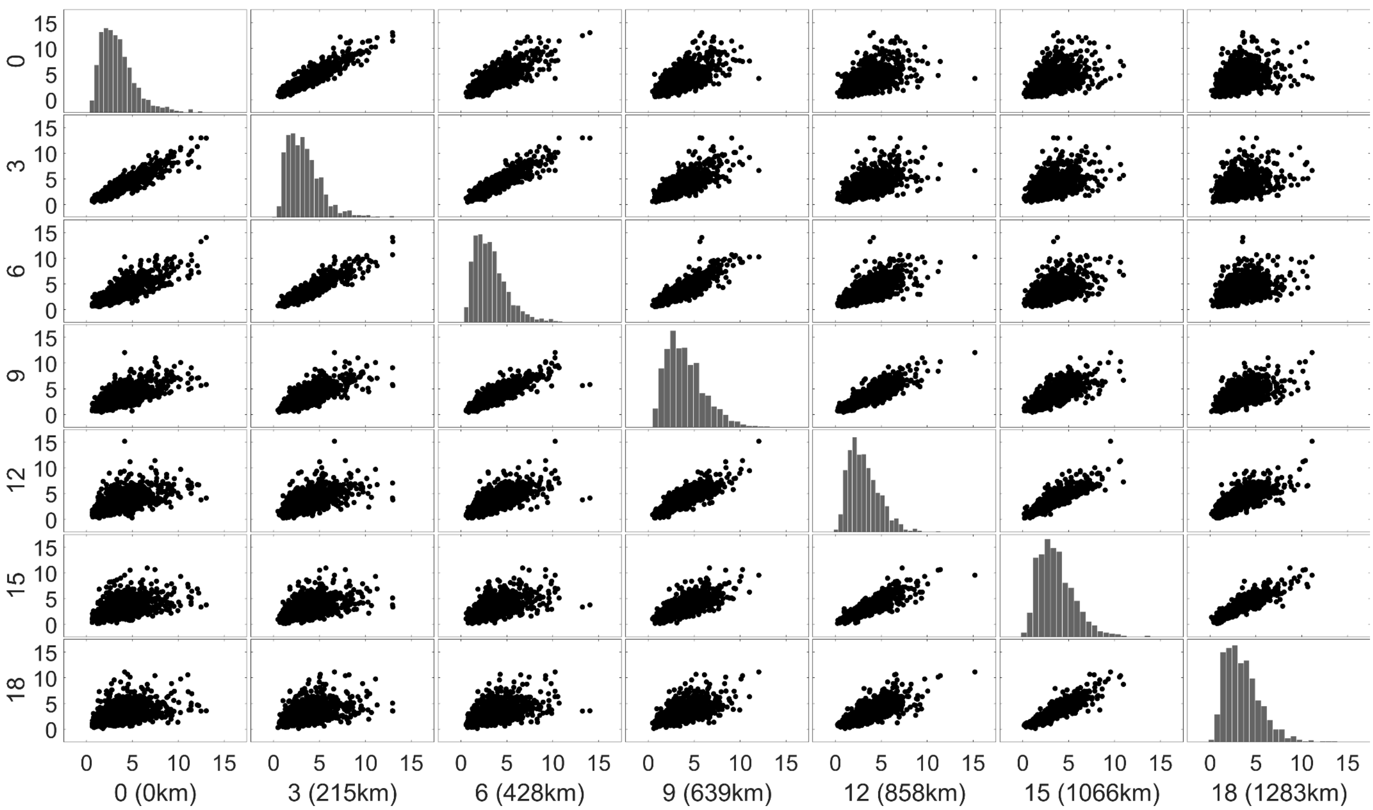

Figure 2. Histograms and scatter plots of H_S measurements for the SWNE transect, on physical scale. Outer axis labels indicate locations (only data for locations 0, 3, 6, ..., 18 are shown). Distances from reference location (in kilometres, km) given in the outer x-axis labels. Inner axis labels are value of H_S (in metres, m).

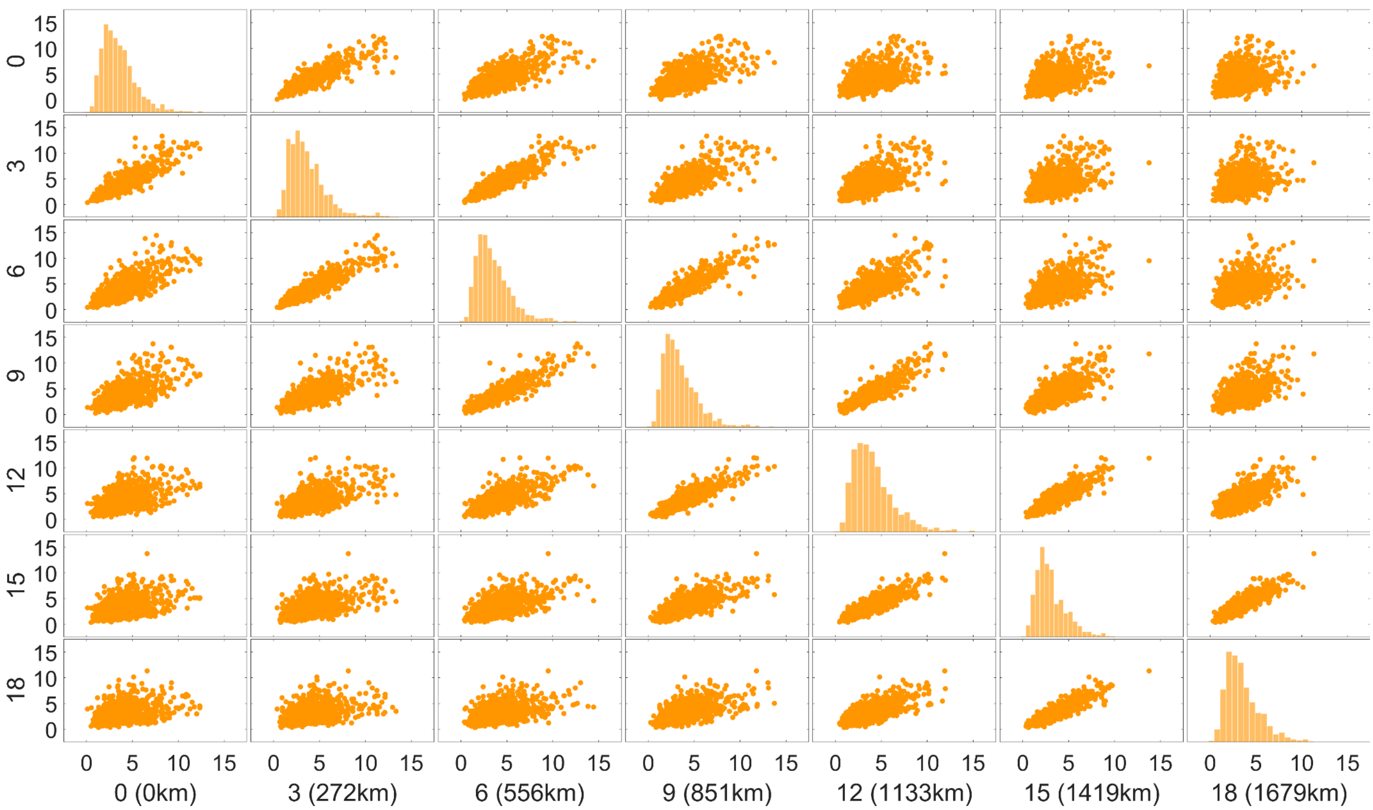

Figure 3. Histograms and scatter plots of H_S measurements for the NWSE transect, on physical scale. Outer axis labels indicate locations (only data for locations 0, 3, 6, ..., 18 are shown). Distances from reference location (in km) given in the outer x-axis labels. Inner axis labels are value of H_S (in m).

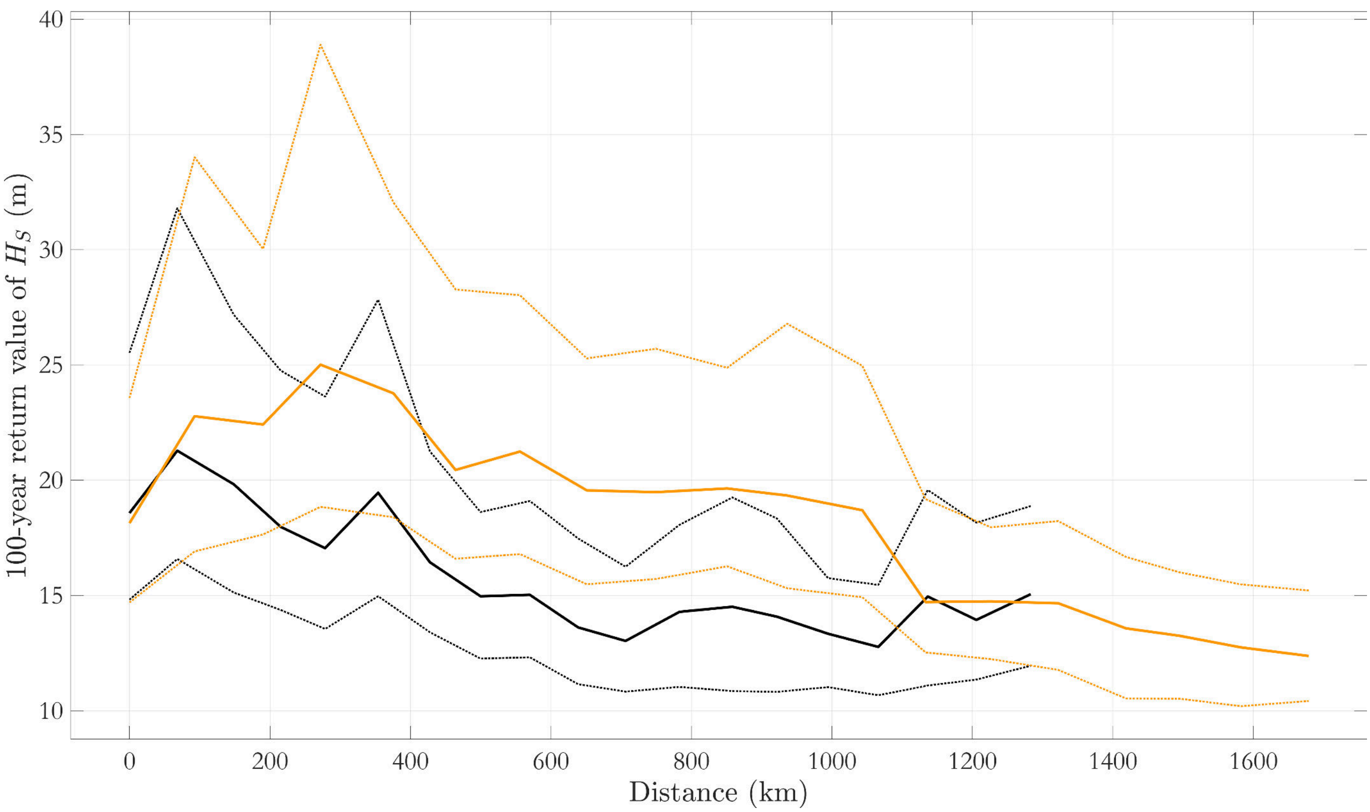

Figure 4. Marginal 100-year return value (m) of H_S for SWNE (black) and NWSE (orange) transects as a function of distance from reference location, in terms of bootstrap median (solid) and bootstrap 95% uncertainty interval (dotted).

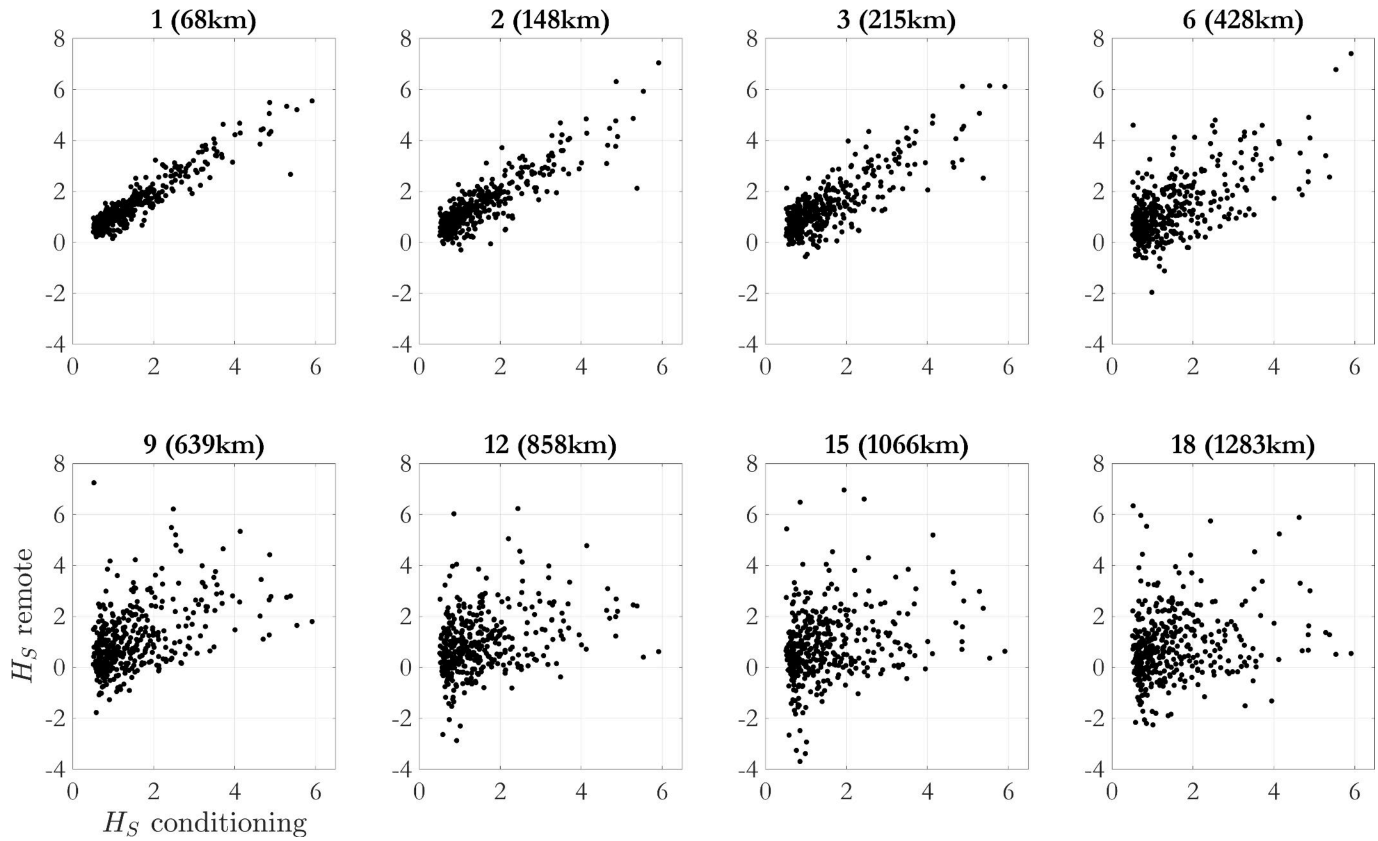

Figure 5. Scatter plots of Laplace-scale H_S data for SWNE transect. Plot of H_S at remote location (y-axis) on H_S at conditioning location (location 0, x-axis) for remote locations 1, 2, 3, 6, 9, 12, 15 and 18 (indicated by panel titles).

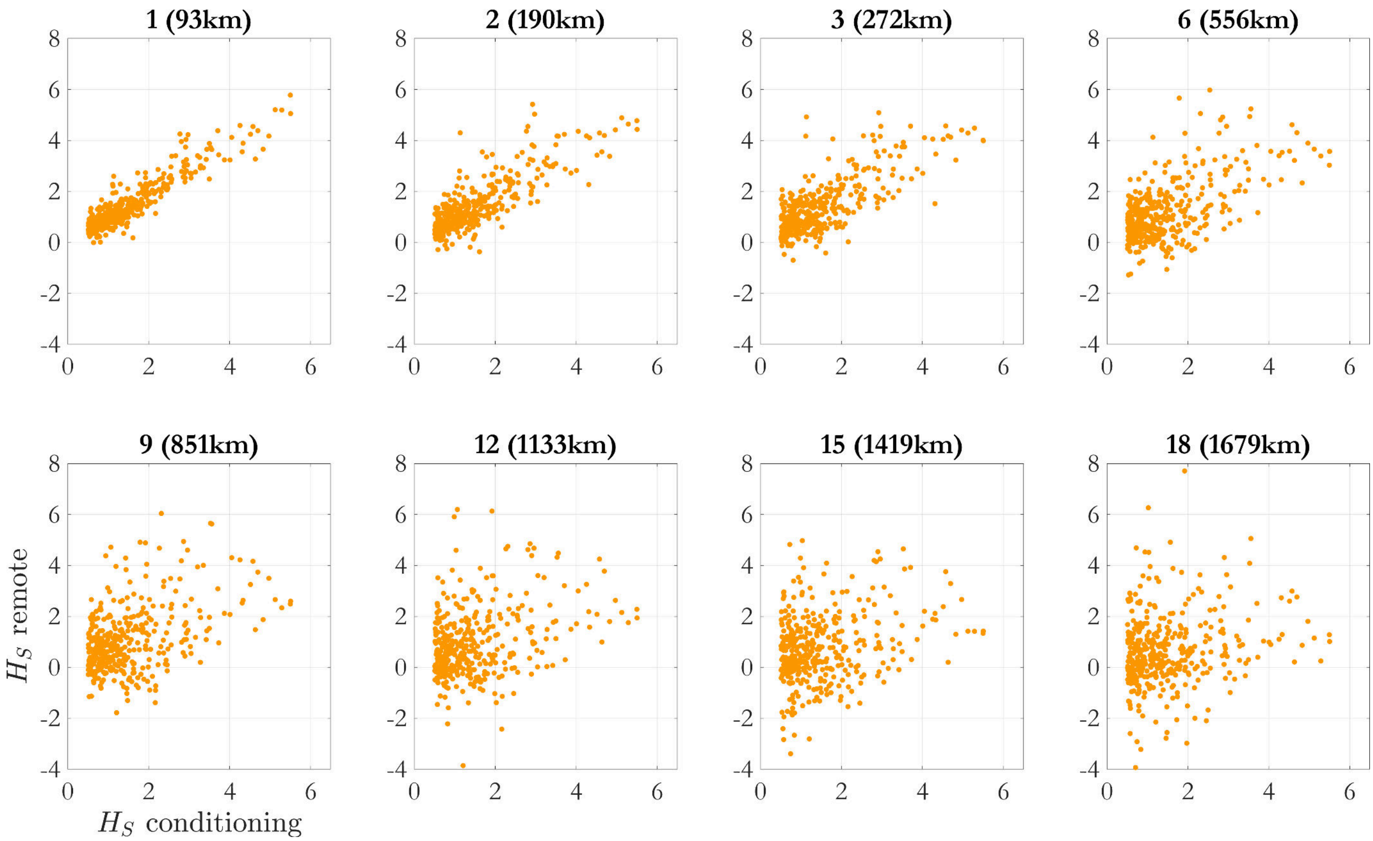

Figure 6. Scatter plots of Laplace-scale H_S data for NWSE transect. Plot of H_S at remote location (y-axis) on H_S at conditioning location (location 0, x-axis) for remote locations 1, 2, 3, 6, 9, 12, 15 and 18 (indicated by panel titles).

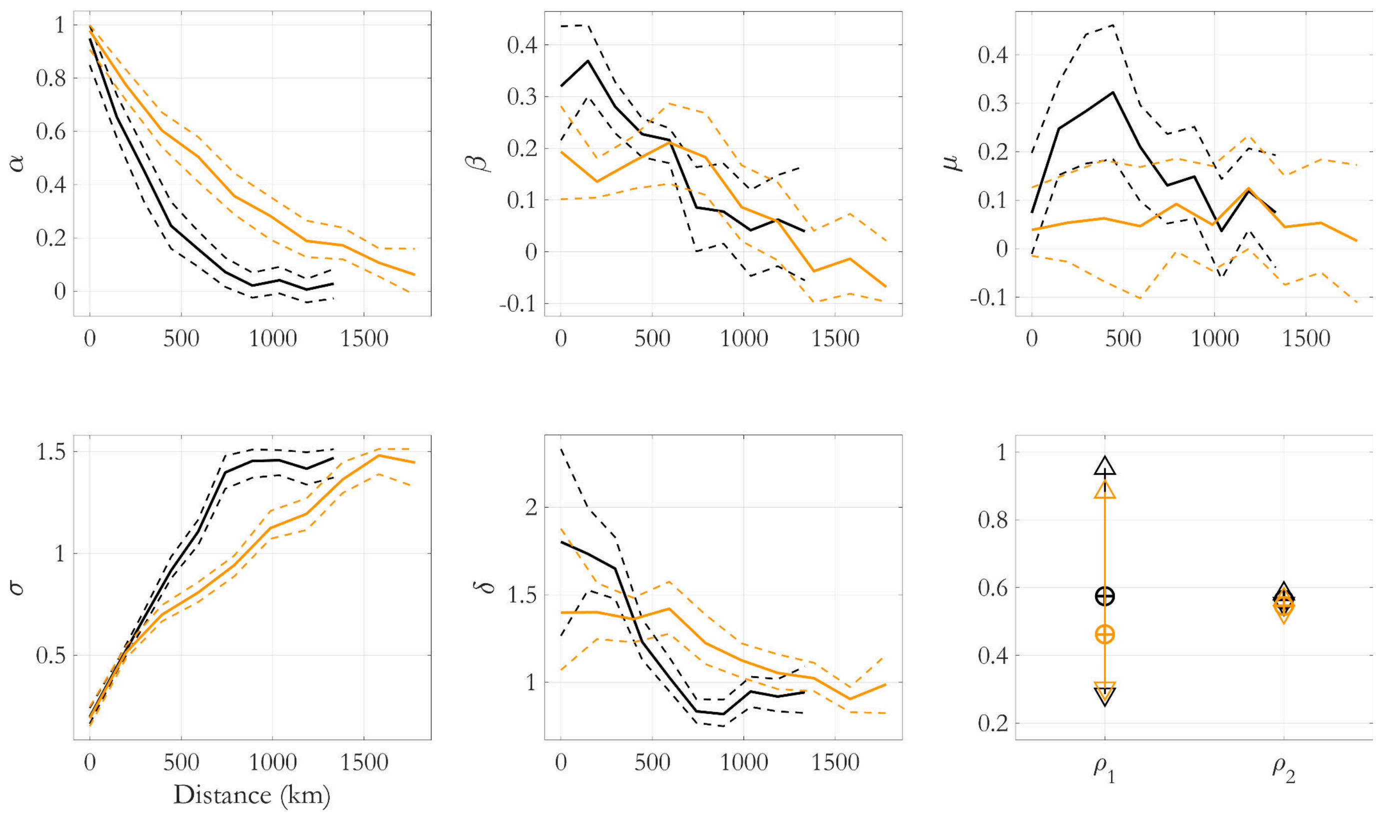

Figure 7. Spatial conditional extremes parameter estimates for the SWNE (black) and NWSE (orange) transects, with conditional quantile constraints imposed. Estimates for α, β, μ, σ and δ as a function of distance (posterior mean and 95% credible interval). Estimates for parameters ρ_1 and ρ_2 in bottom right panel (posterior mean and 95% credible interval).

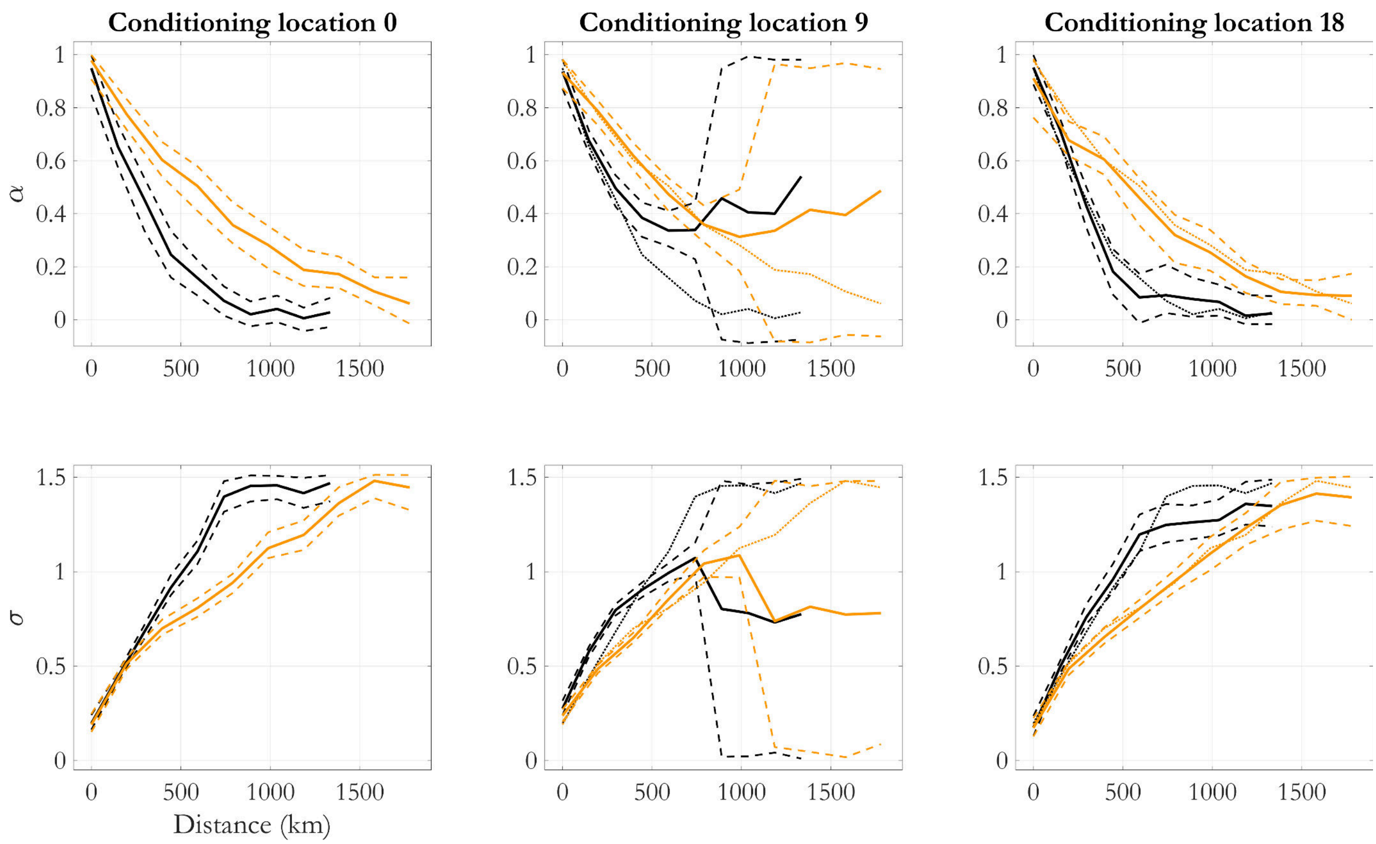

Figure 8. Spatial conditional extremes parameter estimates α (top) and σ (bottom) for the SWNE (black) and NWSE (orange) transects, with conditional quantile constraints imposed, for conditioning locations 0 (left), 9 (centre) and 18 (right). Estimates are shown as a function of distance (displaying the posterior median and associated 95% credible interval). Dotted lines in the centre and right-hand panels correspond to posterior median profiles for conditioning location 0 for comparison. Illustrations of other parameter estimates are available at Shooter et al. (2020a).

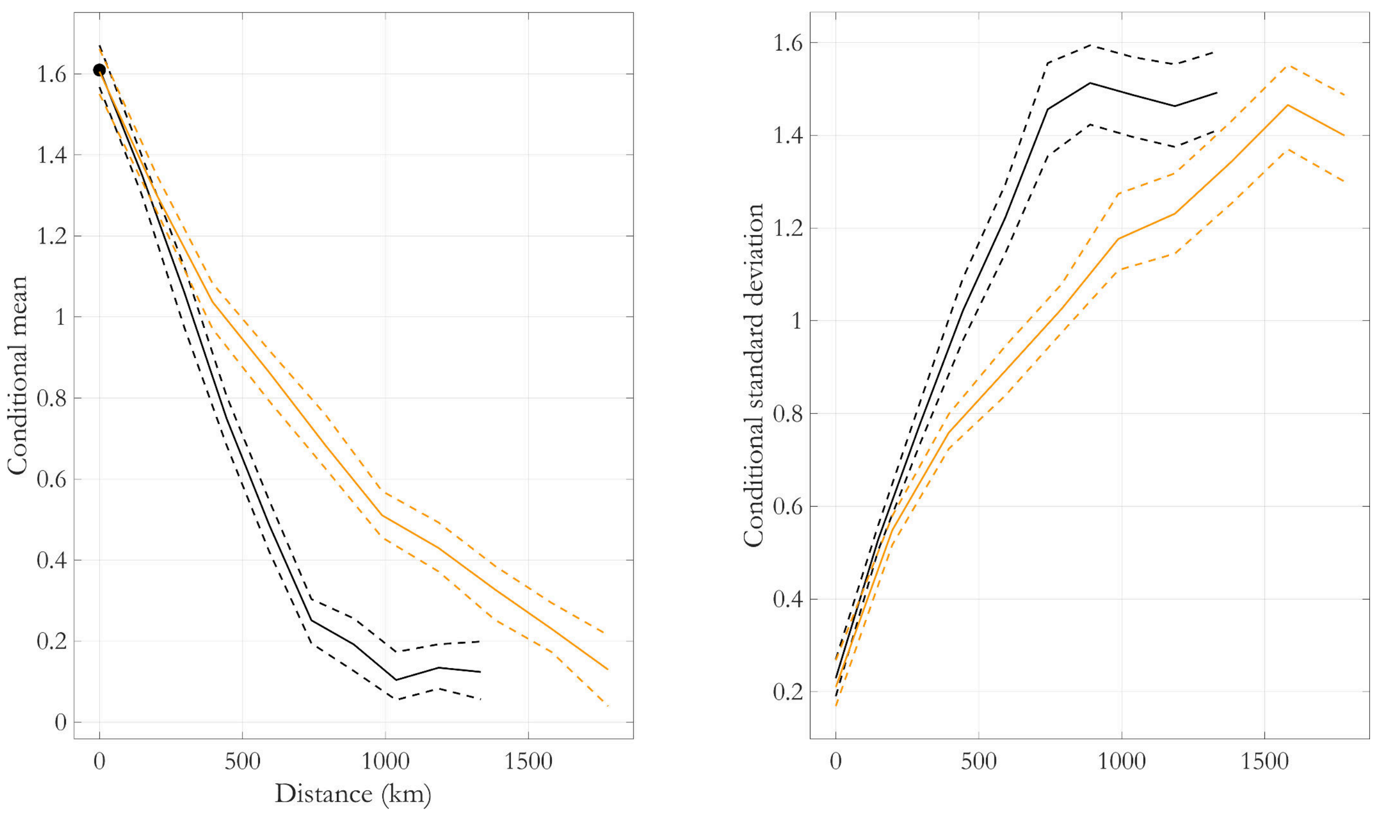

Figure 9. Estimates for Laplace-scale conditional mean m and standard deviation s with distance for SWNE (black) and NWSE (orange) transects, with conditional quantile constraints imposed. Conditioning x_0 has non-exceedance probability 0.9. Solid lines represent posterior medians, with dashed lines representing the upper and lower limits of empirical 95% posterior credible intervals.

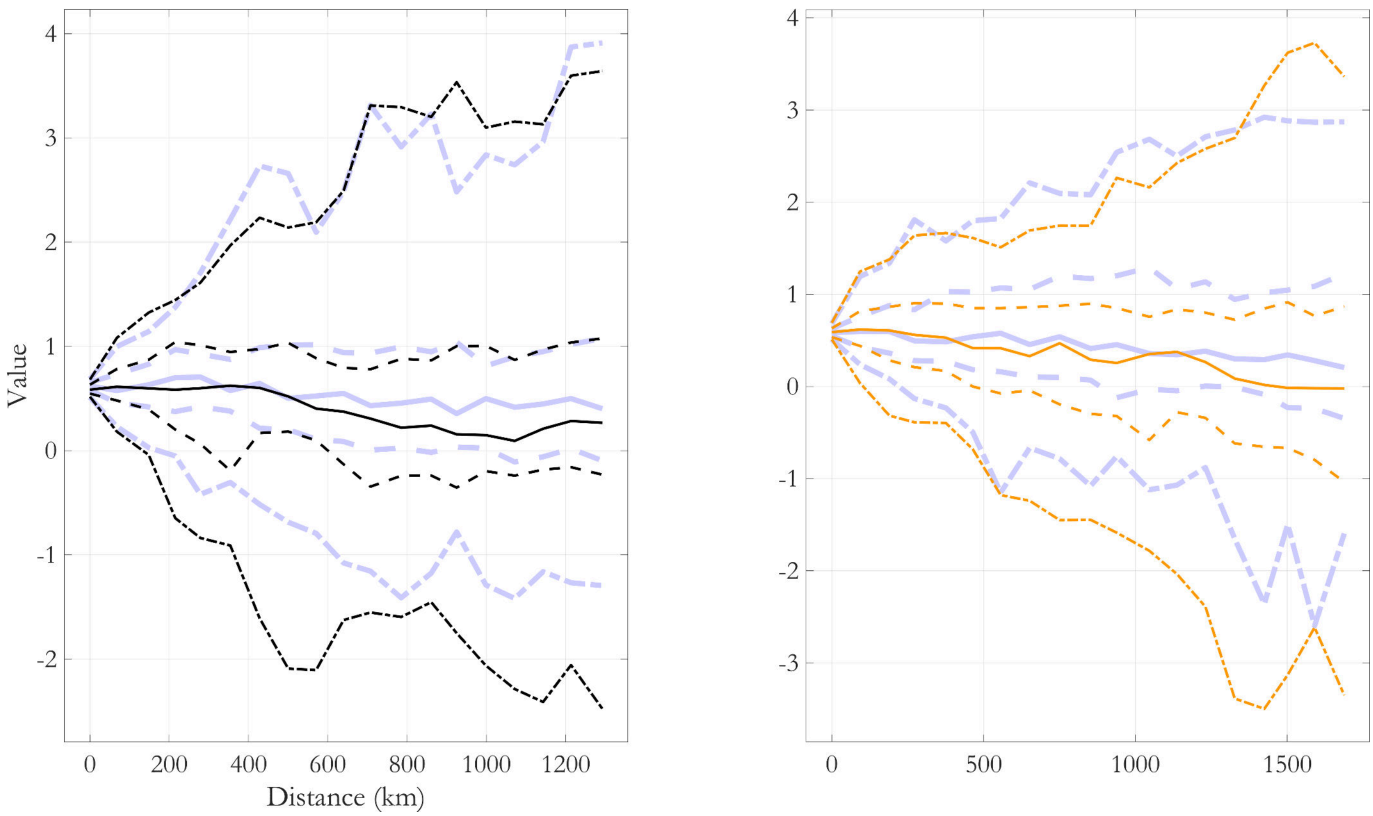

Figure 10. Laplace-scale quantiles of observed trajectories (thick blue) and corresponding trajectories simulated under the estimated spatial conditional extremes model with conditional quantile constraints imposed, for SWNE (black, left) and NWSE (orange, right) transects. Conditioning values x_0 correspond to non-exceedance probabilities $\in (0.7, 0.75]$. Quantile probabilities shown are 0.025, 0.25, 0.5, 0.75, 0.975.

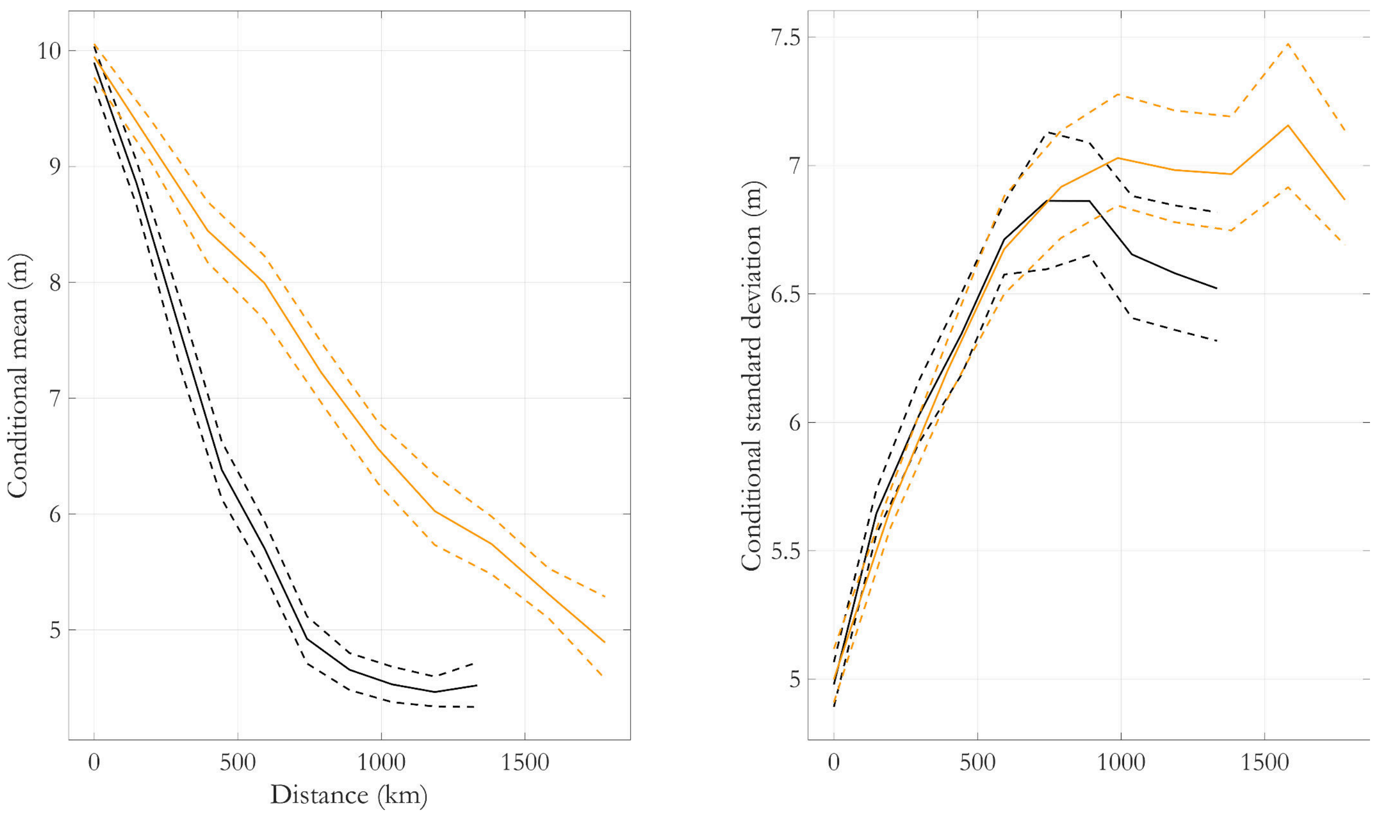

Figure 11. Estimates for physical-scale conditional mean m and standard deviation s with distance for SWNE (black) and NWSE (orange) transects, with conditional quantile constraints imposed. Conditioning value $x_0 = 10$m corresponds to a non-exceedance probability of approximately 0.98 at the reference location on both transects. Solid lines represent posterior medians, with dashed lines representing the upper and lower limits of empirical 95% posterior credible intervals.

TABLES

Altimeter	Transect	Start date (UTC)	End date (UTC)	Number of transects used
JASON-1	SWNE	15-Jan-2002 17:14:52	20-Jun-2013 04:40:28	888
JASON-2	SWNE	04-Jul-2008 14:13:15	16-Jul-2018 10:02:12	696
JASON-3	SWNE	12-Feb-2016 22:33:12	16-Jul-2018 09:17:51	155
JASON-1	NWSE	15-Jan-2002 21:08:22	19-Jun-2013 10:11:39	885
JASON-2	NWSE	05-Jul-2008 20:28:01	13-Jul-2018 17:02:20	693
JASON-3	NWSE	14-Feb-2016 04:48:27	15-Jul-2018 16:44:11	158

Table 1. Description of the JASON transects considered. 'SWNE' refers to the normal ascending transect. 'NWSE' refers to the opposite descending transect.