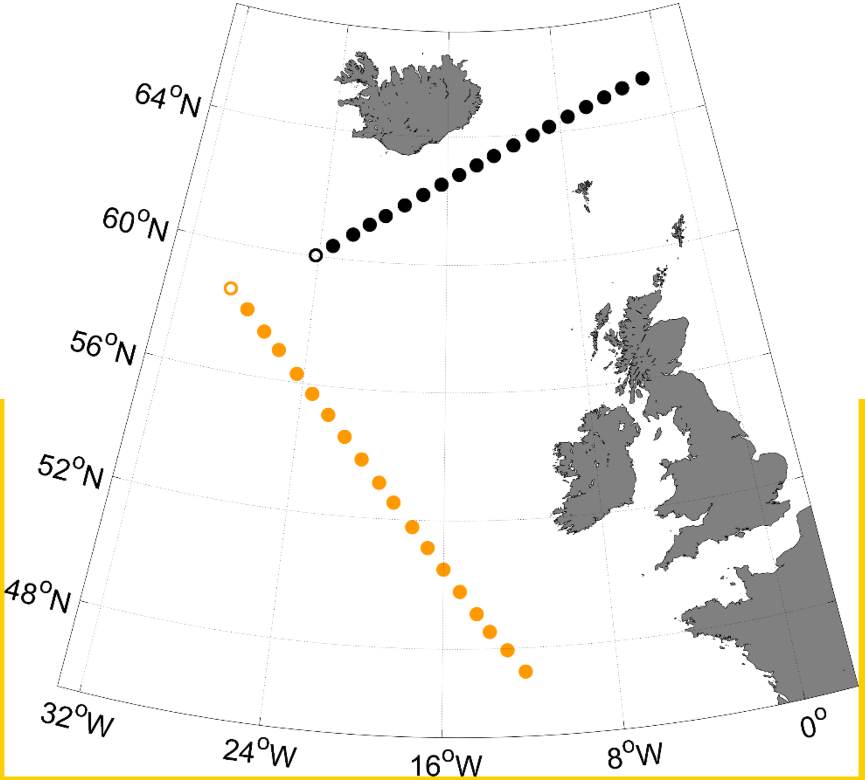
Spatial dependence of extreme seas using satellite altimeter measurements

in the North East Atlantic

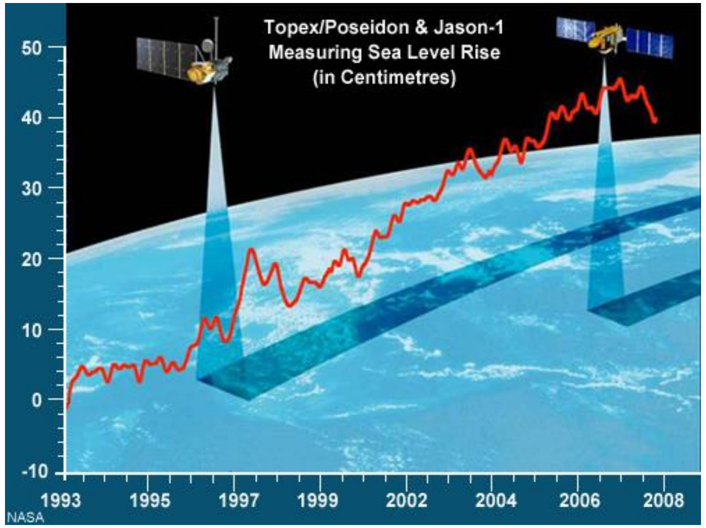


Emma Ross (Shell Global Solutions International B.V., The Netherlands)
Philip Jonathan (Shell Research Limited, Lancaster University, UK)
Rob Shooter (Met Office, UK)
Agustinus Ribal (Dept. of Maths, Hasanuddin University, Indonesia)
Ian Young (Dept of Infrastructure Engineering, The University of Melbourne, Australia)

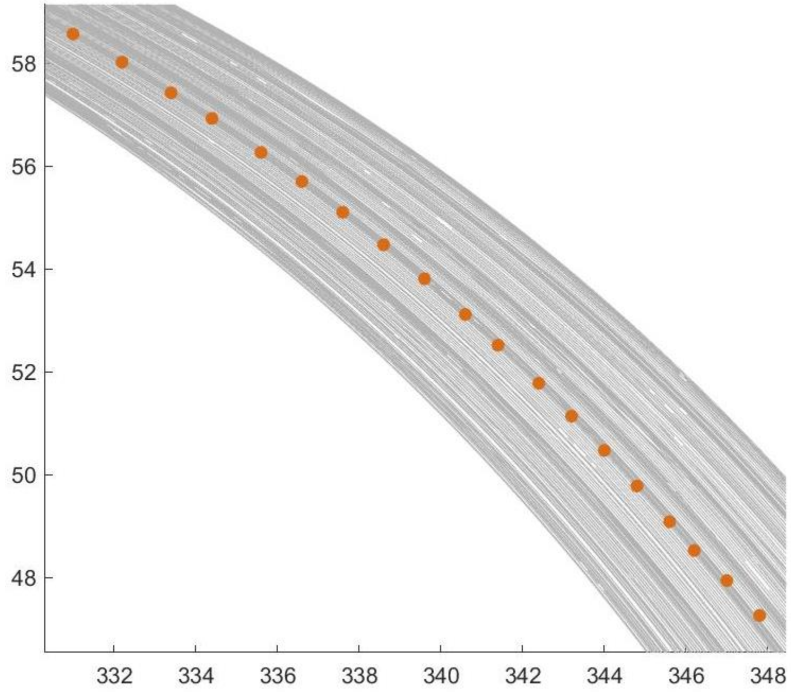
Altimeter Data Pre-Processing

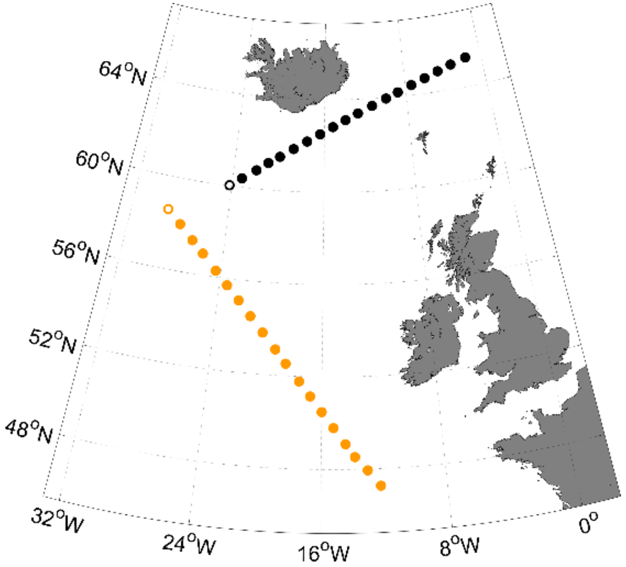
- JASON (Joint Altimetry Satellite Oceanography Network) 1, 2 and 3 altimeter measurements of significant wave-height (Hs)
- Calibrated against buoy data and quality controlled as in *Ribal and Young (2019)*
- Approx. timespan:

JASON-1:	2002 - 2013
JASON-2:	2008 - 2018
JASON-3:	2016 - 2018.



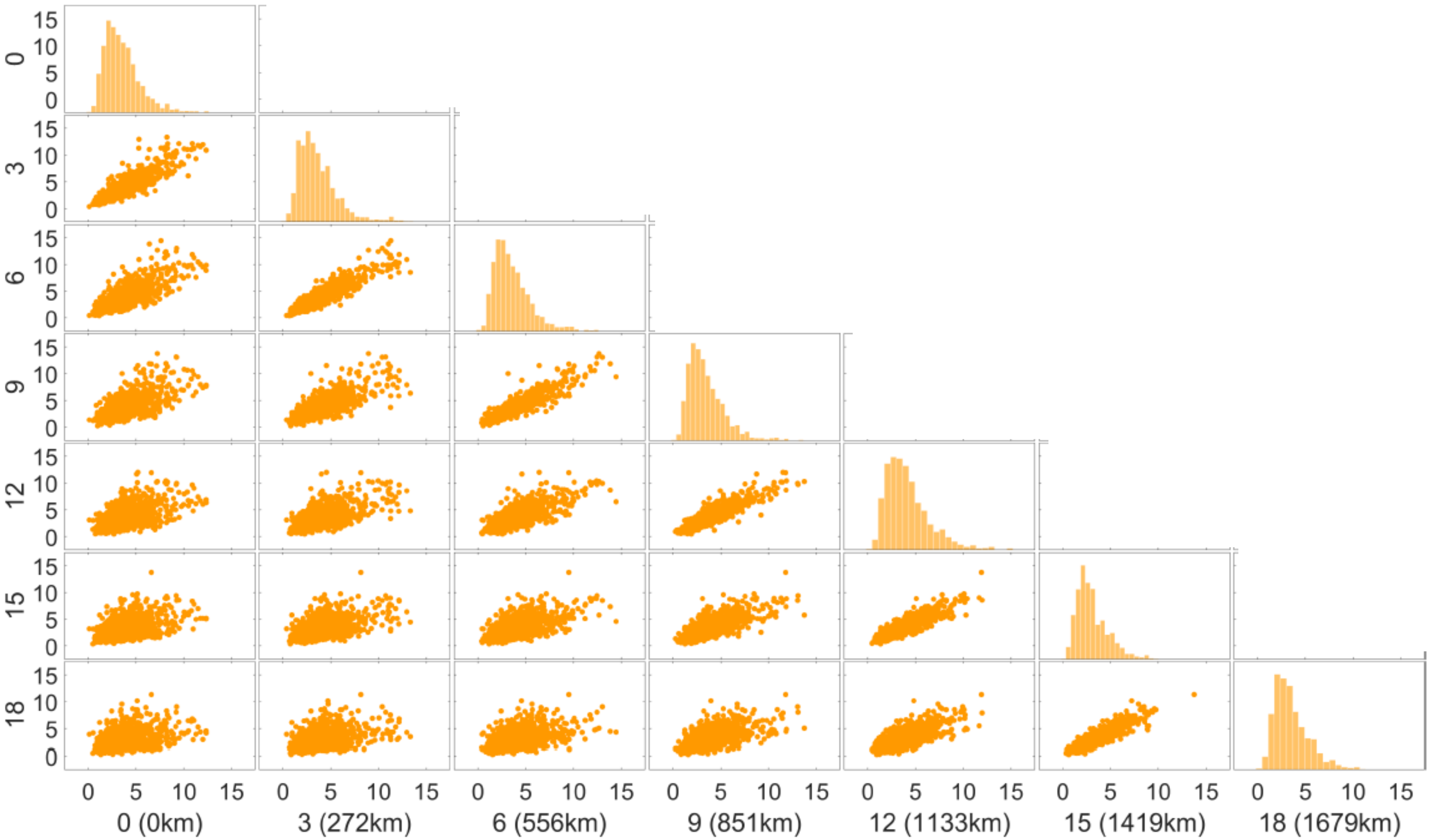
www.eumetsat.int/jason



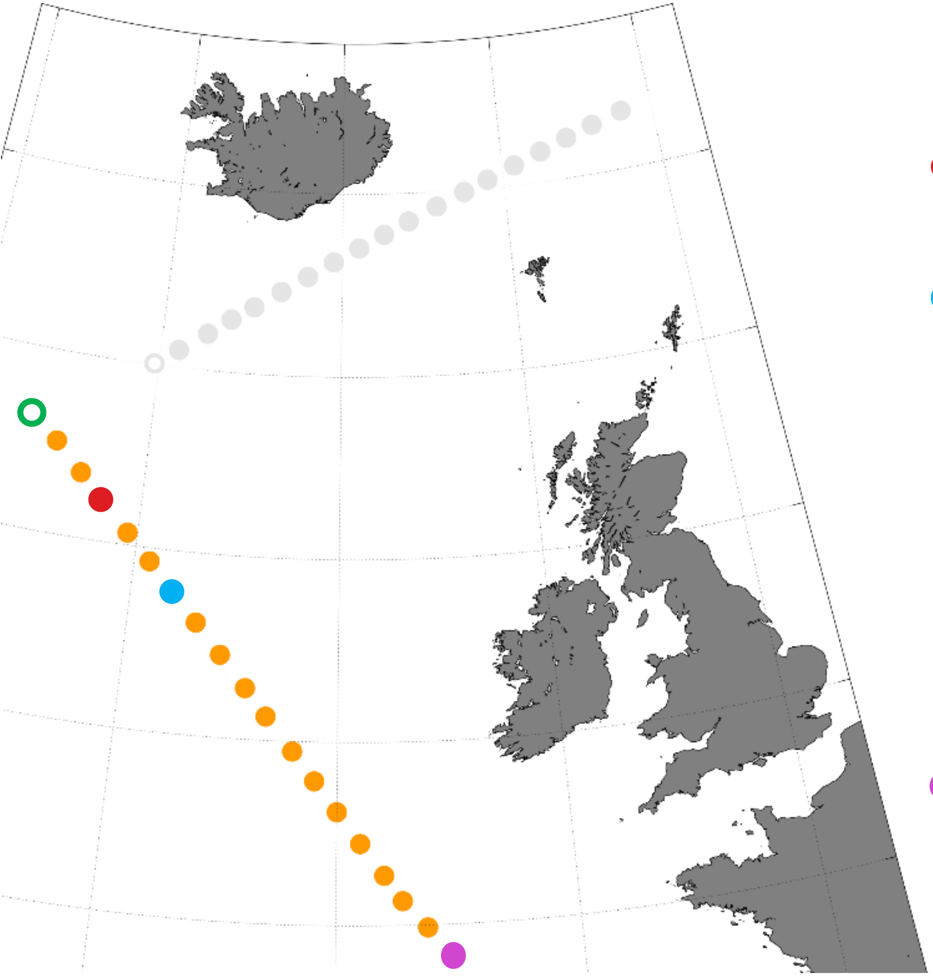


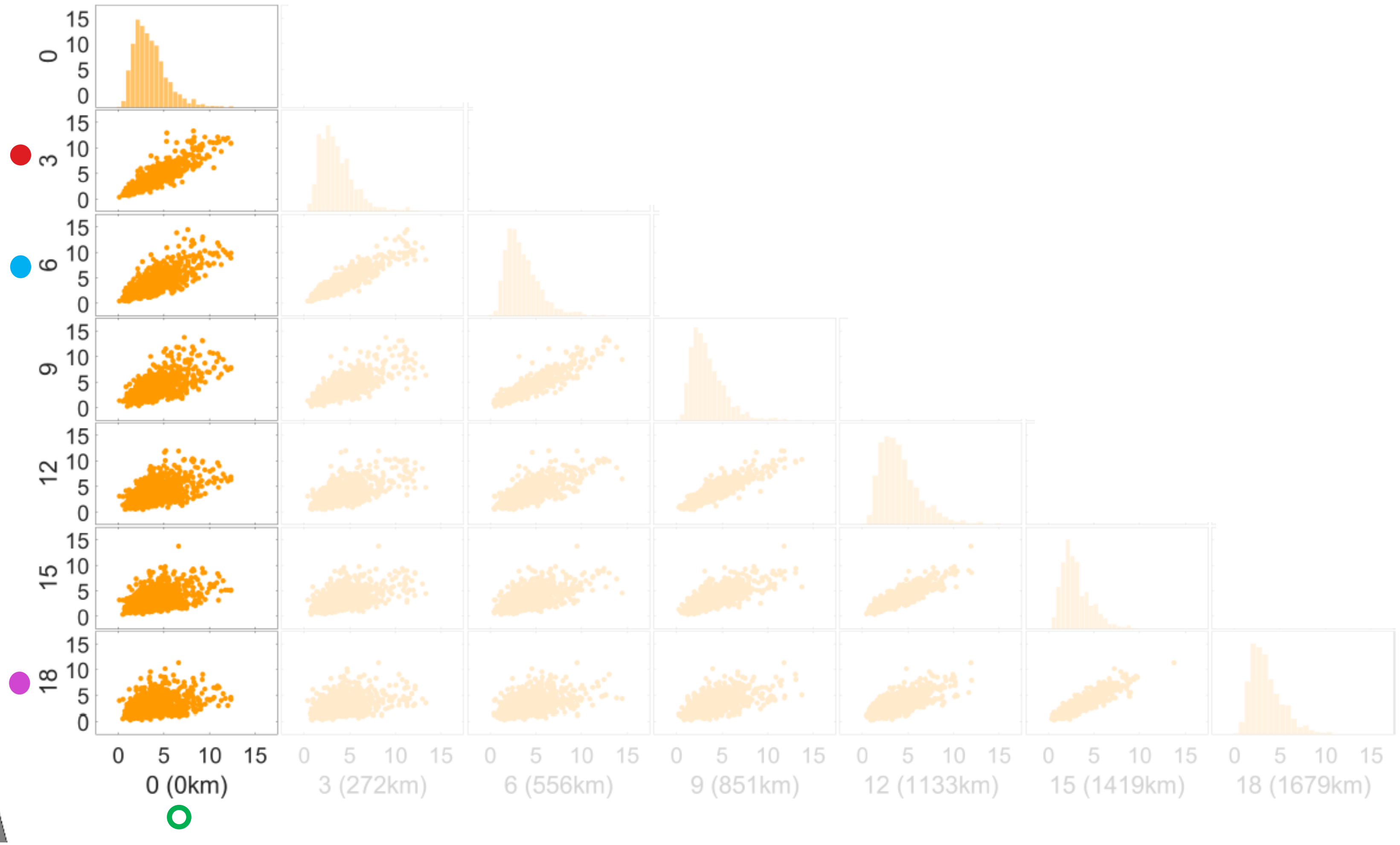
- Define *registration locations* which mark out a template transect
- For each satellite pass, find the nearest point on the transect to each of the registration locations. If distance > 50km, pass not registered.
- Result: for each registration location, a sample of Hs observations from different satellite passes.

Hs-Samples at Reference Locations

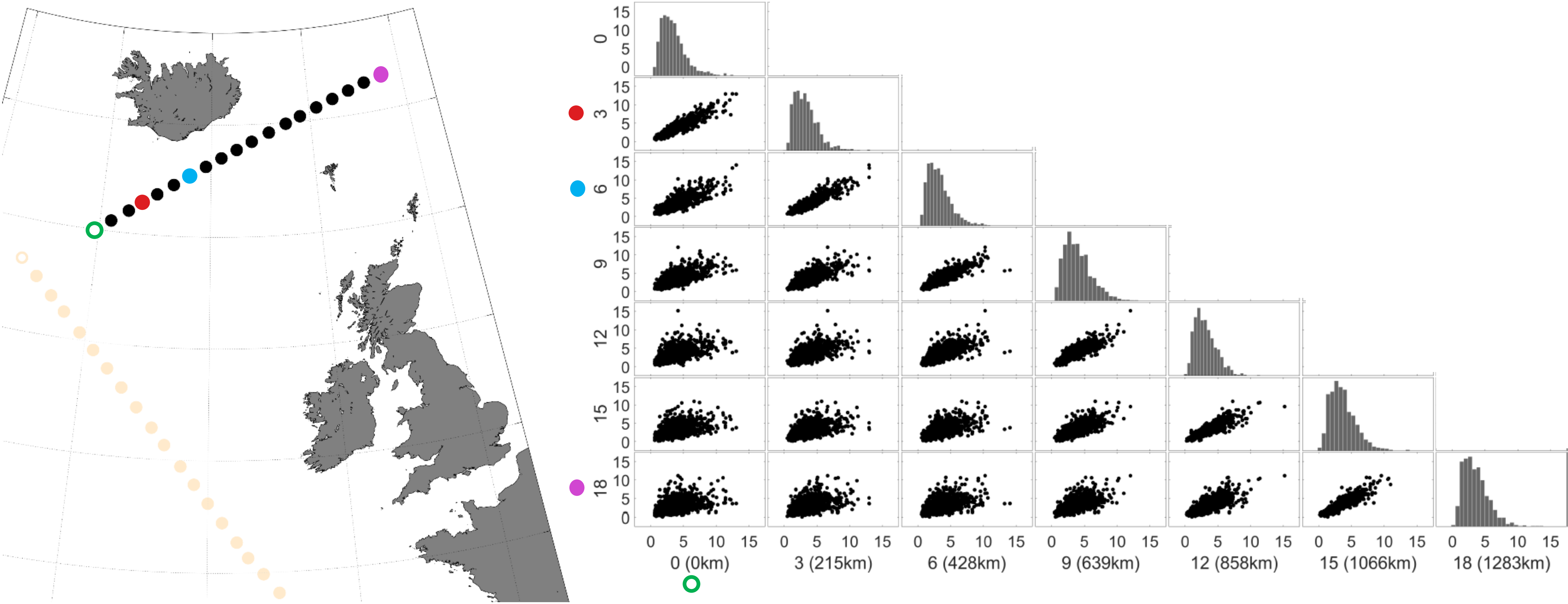


Hs-Samples at Reference Locations





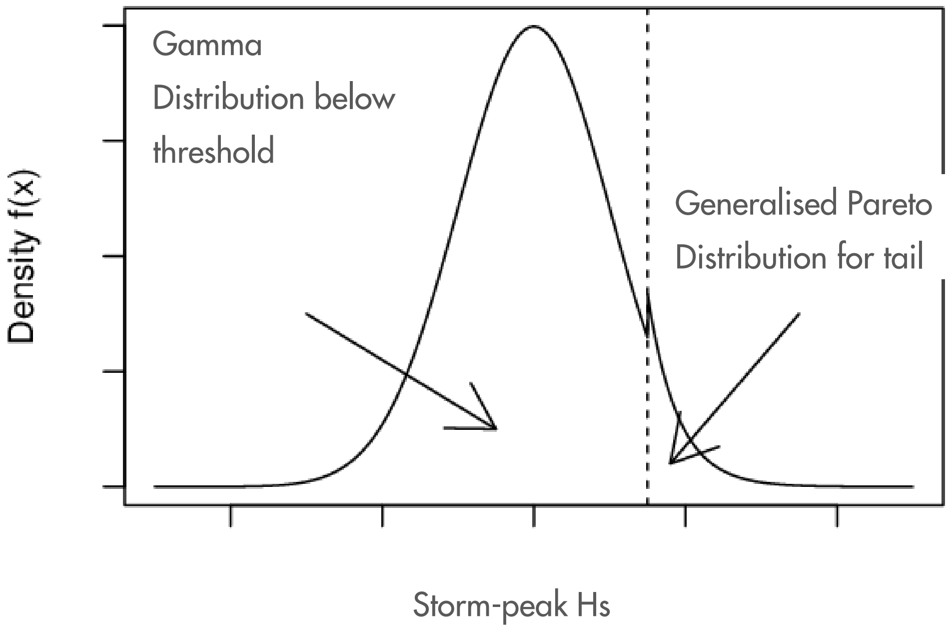
Hs-Samples at Reference Locations



Marginal Modelling

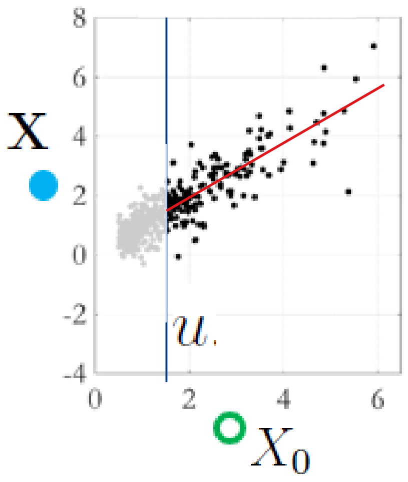
- Conditional extremes model requires **transformation of observations to standard marginal** (Laplace or Gumbel) scale prior to analysis
- **Generalised Pareto** model fitted to tail using maximum likelihood estimation at each registration location independently
- **Probability Integral Transform** then used to transform to **Laplace scale**

- No attempt made here to account for effects of covariates such as storm direction and season, which have been found to be influential in marginal extreme value inference (Feld et al. (2015)) – further work.



Conditional Extremes Model

Heffernan & Tawn (2004)



$$\mathbf{X}|\{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\beta}\mathbf{Z}$$

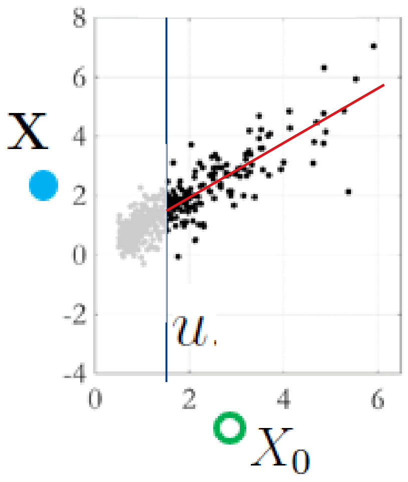
Where:

- X_0 = **Laplace-scale** Hs at conditioning location

  $\mathbf{X}$ = Laplace-scale Hs at remote location
- $x_0 > u$: some sufficiently high threshold
- $\beta \in (-\infty, 1]$ we assume +'ve dependence so $\alpha \in [0, 1]$
- $\mathbf{Z}$ is independent of X_0

Conditional Extremes Model

Heffernan & Tawn (2004)

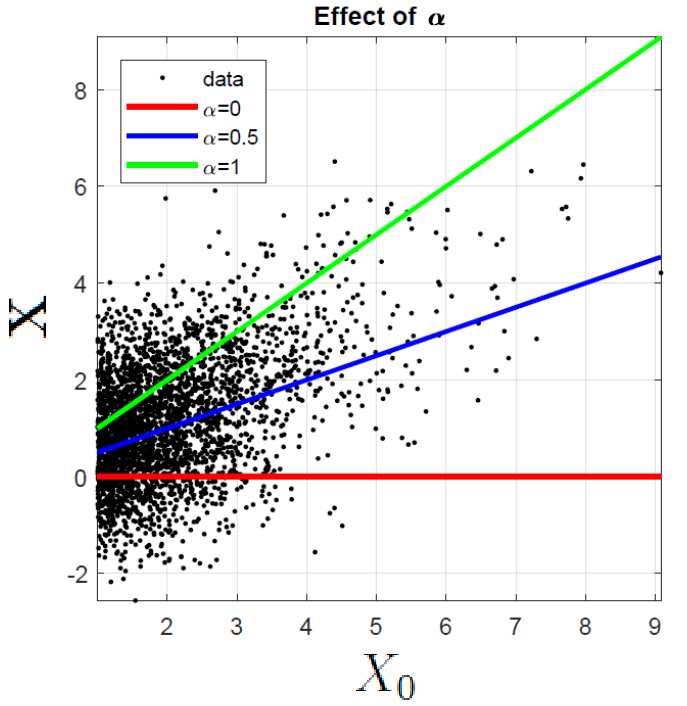


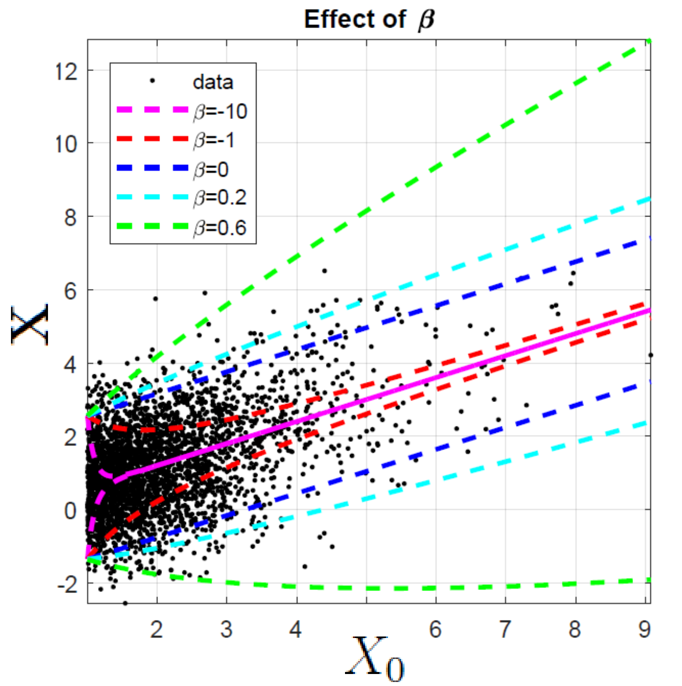
$$\mathbf{X} | \{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\beta} \mathbf{Z}$$

Where:

- X_0 = Laplace-scale Hs at conditioning location

  $\mathbf{X}$ = Laplace-scale Hs at remote location
- $x_0 > u$: some sufficiently high threshold
- $\beta_j \in (-\infty, 1]$ we assume +'ve dependence so $\alpha \in [0, 1]$
- $\mathbf{Z}$ is independent of X_0





- Asymptotic **dependence** when $\alpha = 1$
- Asymptotic **independence** when $\alpha \in (0, 1)$
- Perfect independence when $\alpha = 0$

Spatial Conditional Extremes Model

Shooter et. Al (2019), Wadsworth and Tawn (2019)

$$(X_1, \dots, X_q) | \{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\boldsymbol{\beta}} \mathbf{Z}$$

Where:

- $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are now vectors of length q (total number of remote locations), with each element $\alpha_j = \alpha(d_j)$ a function of the ***distance*** between location j and the conditioning location 0
- $\mathbf{Z} \sim \mathrm{DL}_q(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{\delta}; \Sigma)$ i.e. has delta-Laplace (generalized Gaussian) margins with parameters μ, σ and δ which depend on distance d , and:
- Σ is the $q \times q$ correlation matrix for a **conditional** Gaussian dependence structure between residual components, with parameters $\rho_1, \rho_2 \in \mathbb{R}_{>0}$

- Bayesian inference to estimate the joint posterior distribution of the SCE model parameters $\Omega = \{\{\alpha_j, \beta_j, \mu_j, \sigma_j, \delta_j\}_{j=1}^q, \rho_1, \rho_2\}$

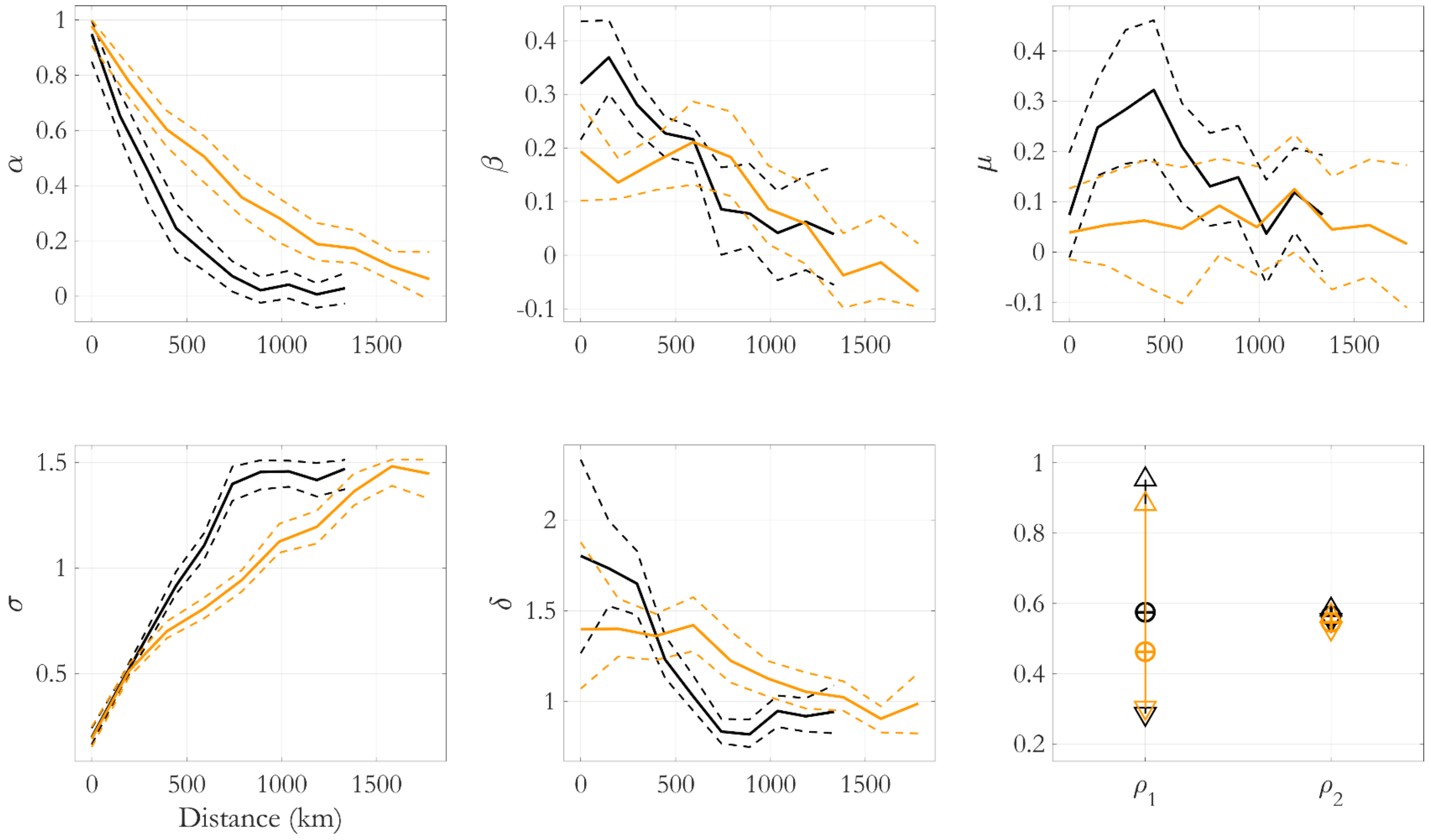
Summary of Procedure

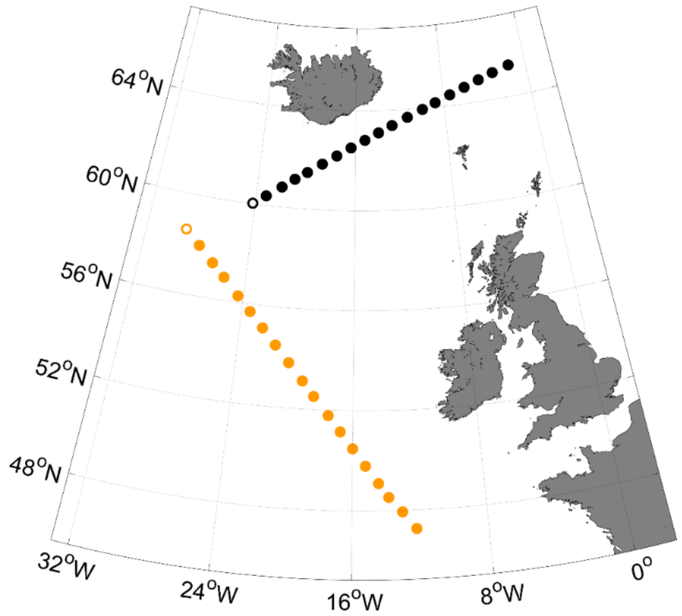
1. **Pick a set of reference locations** along a template transect, and find nearest observations from each satellite pass (within 50km)
2. **Fit generalized Pareto distributions** to the tail of the resulting Hs-samples for each reference location (marginal modelling)
3. For each ref. location, **transform** GP-distributed Hs data **to standard Laplace scale**
4. **Fit Spatial Conditional Extremes** (SCE) model, e.g. using Adaptive MCMC procedure
5. **Interpret** results – using the fitted SCE model we can:
   a) Establish Asymptotic Dependence/Independence from resulting fitted-parameters
   b) Use fitted-model to establish conditional return values at different distances; and to simulate the evolution of Hs along the template transect
   c) Compare results for different template transects

github.com/ygraigarw/SpatialConditionalExtremesSatellite
github.com/ECSADES/ecsades-matlab

Analysis on NE Atlantic Transects

Fitted Model Parameters



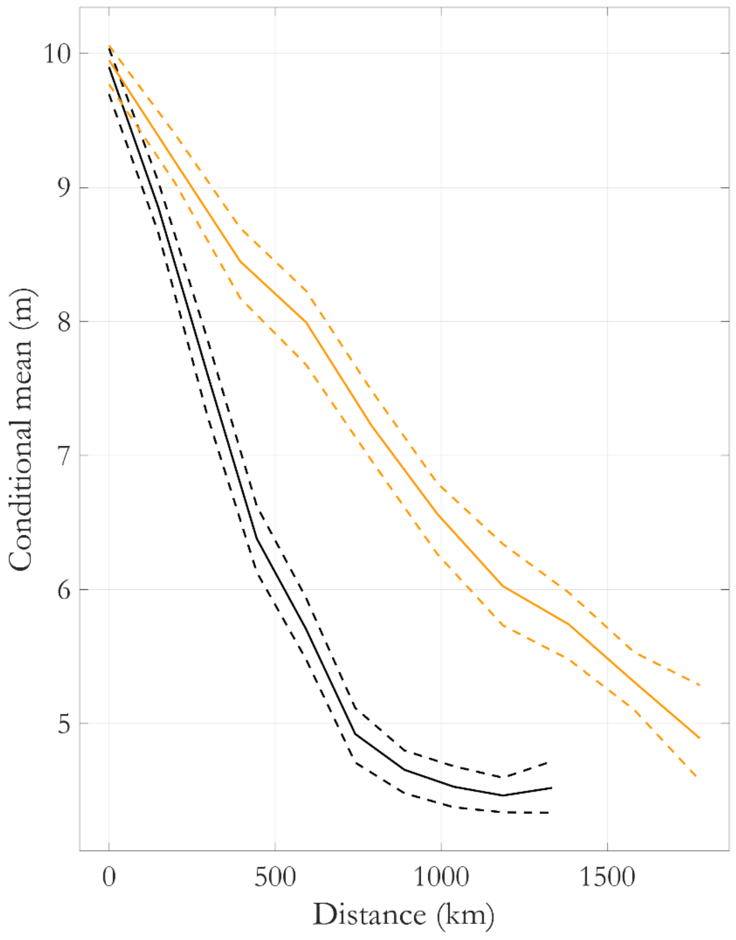


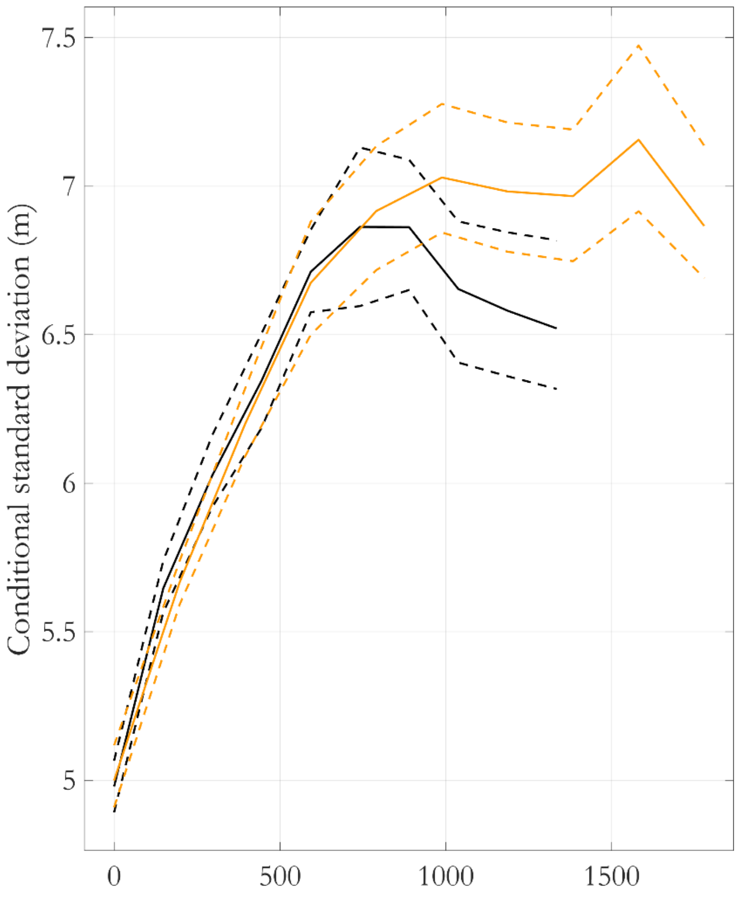
$$(X_1, \ldots, X_q)|\{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\boldsymbol{\beta}} \mathbf{Z}$$

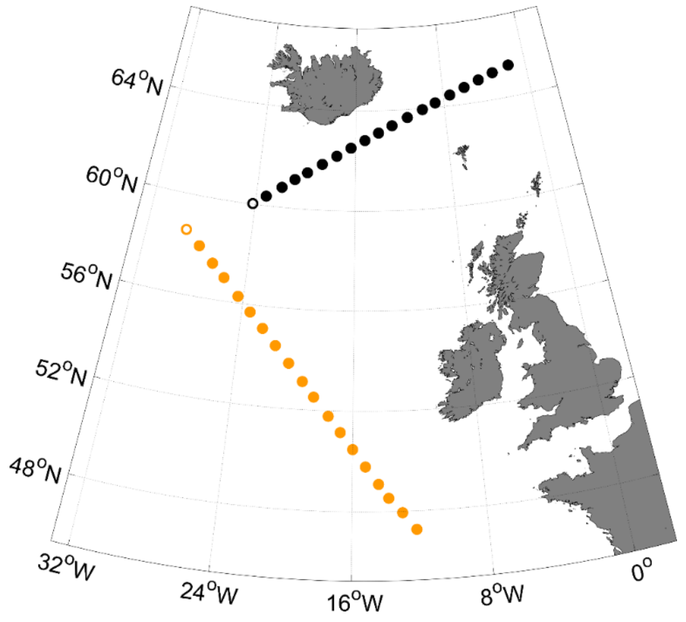
$$\mathbf{Z} \sim \mathrm{DL}_q(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{\delta}; \boldsymbol{\Sigma})$$

Analysis on NE Atlantic Transects

Conditional Mean and Standard Deviation

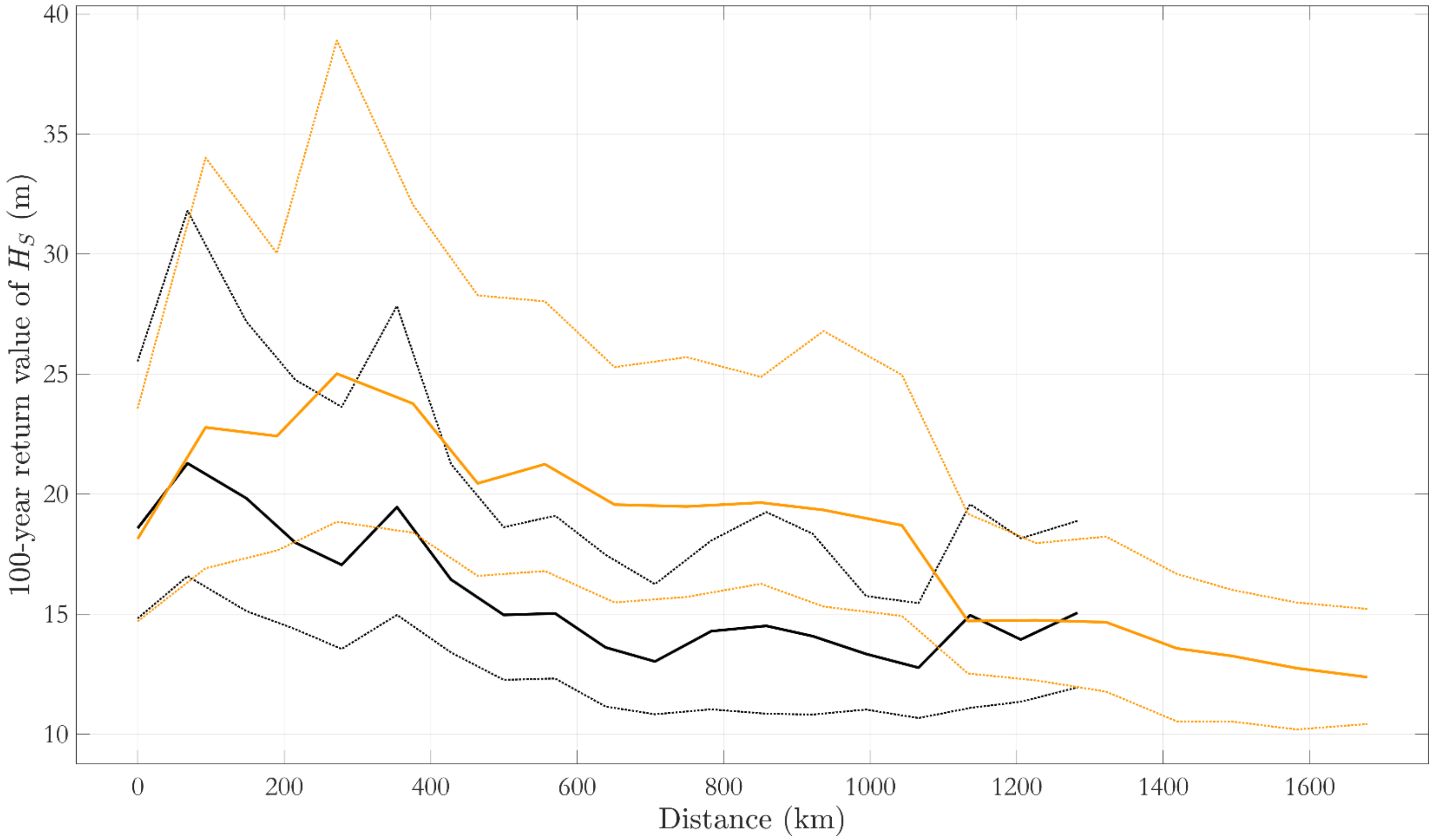


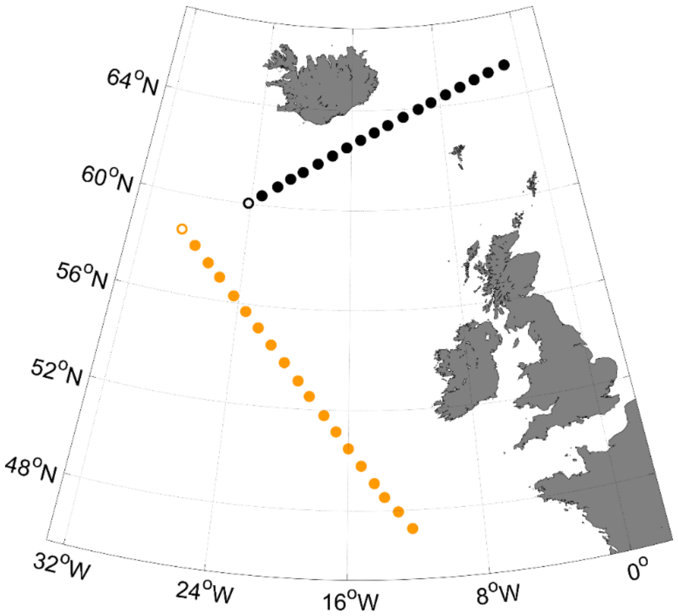




Analysis on NE Atlantic Transects

100-year Return Values





Similar to 100-year return values for Hs reported in Takbash et al. (2019)

Summary & References

- **Conditional Spatial Extremes model** used to quantify spatial dependence of extreme values of altimeter measurements of Hs in the NE Atlantic.
- Model **accommodates both asymptotic-independence and -dependence** whilst being relatively straightforward to implement
- Strong evidence of **differences in extremal spatial characteristics** along the different transects.
- **Altimeter measurements** provide a useful high-quality resource for examination of spatial structure of wave fields.

References

- Details of this work in paper submitted to Environmetrics: lancs.ac.uk/~jonathan/
- Heffernan and Tawn (2014), Feld et al. (2015), Kereszturi et al. (2016), Tawn et al. (2018), Wadsworth and Tawn (2019), Shooter et al. (2020b), Takbash et al. (2019)
- MATLAB repositories:

*github.com/ygraigarw/**SpatialConditionalExtremesSatellite***

github.com/ECSADES/ecsades-matlab