Multivariate spatial conditional extremes

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(Slides and draft paper at www.lancs.ac.uk/∼jonathan)

Acknowledgement, motivation, related work

Acknowledgement

- Jon Tawn and Jenny Wadsworth (Lancaster), David Randell (Shell)

Motivation

- How useful are satellite observations of ocean waves and winds?
- Could they become the primary data source for decisions soon?
- What are the spatial characteristics of extremes from satellite observations?

Related work

- Heffernan and Tawn [2004] (CE), Heffernan and Resnick [2007]
- Shooter et al. [2019] (SCE), Wadsworth and Tawn [2019] (SCE)
- Shooter et al. [2021b], Shooter et al. [2021a] (SCE applications)

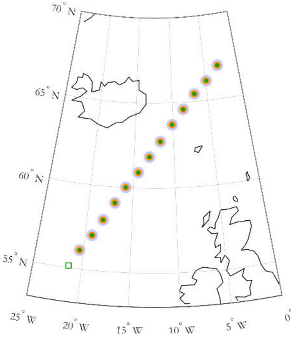
Competitors (= MSPs, hierarchical MSPs and multivariate MSPs)

- Reich and Shaby [2012], Vettori [2017], Vettori et al. [2019]
- Genton et al. [2015], Huser and Wadsworth [2020]

Summary of talk : Outline

- A look at the data
- Brief overview of methodology, extended to multiple fields
- Results for joint spatial structure of extreme scatterometer wind speed, hindcast wind speed and hindcast significant wave height in the North Atlantic
- Implications for future practical applications

Summary of talk : Methodology in nut-shell



- Condition on large value x of first quantity X_{01} at one location $j = 0$
- Estimate "conditional spatial profiles" for $m > 1$ quantities $\{X_{jk}\}_{j=1,k=1}^{p,m}$ at $p > 0$ other locations

$$X_{jk} \sim \text{Lpl}$$

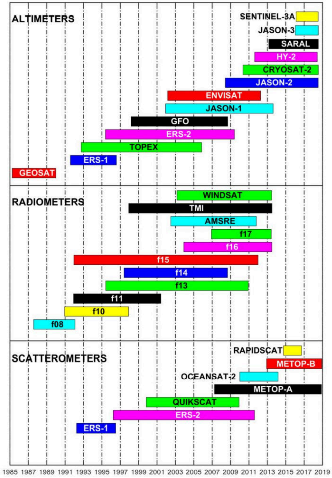
$$x > u$$

$$\boldsymbol{X} | \{X_{01} = x\} = \boldsymbol{\alpha} x + x^{\boldsymbol{\beta}} \boldsymbol{Z}$$

$$\boldsymbol{Z} \sim \text{DL}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{\delta}; \boldsymbol{\Sigma}(\boldsymbol{\lambda}, \boldsymbol{\rho}, \boldsymbol{\kappa}))$$

- MCMC to estimate $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ and $\boldsymbol{\rho}$, $\boldsymbol{\kappa}$, $\boldsymbol{\lambda}$
- $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ spatially smooth for each quantity
- Residual correlation $\boldsymbol{\Sigma}$ for conditional Gaussian field, powered-exponential decay with distance

Satellite observation



[Ribal and Young 2019]

Features

- Altimetry: H_S and U_{10}
- Scatterometry: best for U_{10} and direction
- > 30 years of observations
- Spatial coverage is by no means complete: one observation daily if all well
- Calibration necessary (to buoys and reanalysis datasets, Ribal and Young 2020)
- METOP(-A,-B,-C) since 2007

H_S: significant wave height (m)
U_{10}: wind speed (ms^{-1}) at 10m (calibrated to 10-minute average wind speed)

Hindcast data, objectives

Hindcast data

- Physical simulator calibrated to observations (e.g from buoys)
- NORA10 hindcast covers North Atlantic off UK (Breivik et al. 2013)
- Data available 1957-2018

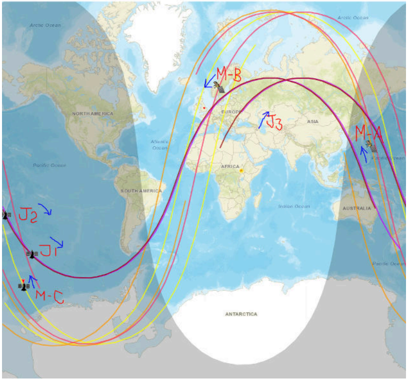
Initial objective

- Joint spatial inferences about extremes using all of
  - H_S (JASON)
  - directional U_{10} (METOP)
  - directional H_S and directional U_{10} (NORA10)
- Not feasible: poor joint spatial coverage of JASON and METOP

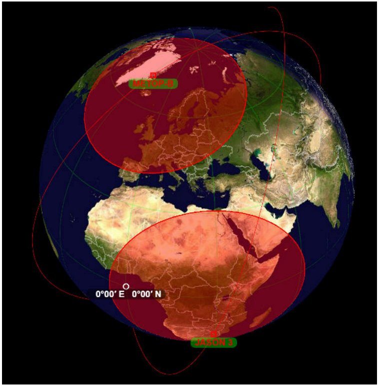
Revised objective

Joint spatial inferences about extremes of directional U_{10} (METOP), hindcast directional H_S and directional U_{10} (NORA10)

JASON and METOP



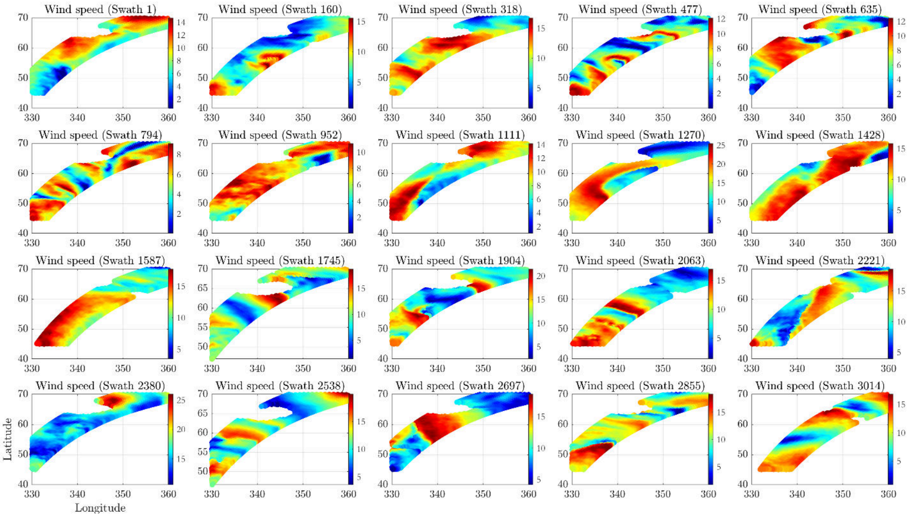
[n2yo.com, accessed 06.09.21 at around 1100UK]



[stltracker.github.io, accessed 27.08.2021 at around 1235UK]

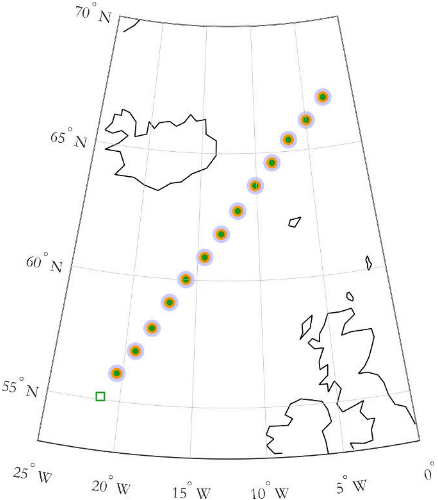
- JASON and METOP similar polar orbits
- JASON all ascending, METOP all descending over North Atlantic
- Joint occurrence of JASON and METOP over North Atlantic rare

Swath wind speeds



Daily descending METOP swaths. Satellite swath location changes over time. Spatial structure evident

Registration locations

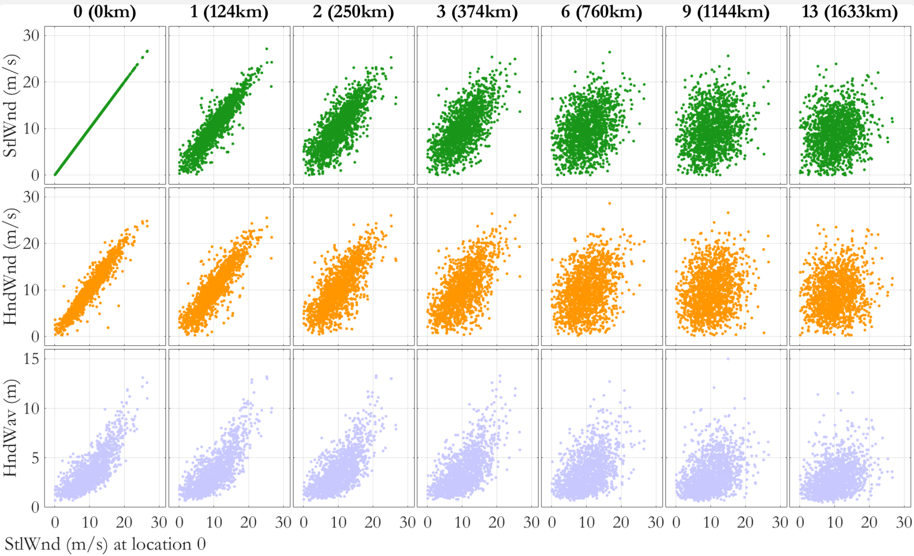


Registration locations : square is conditioning location
StlWnd (green), HndWnd (orange), HndWav(blue)

Procedure

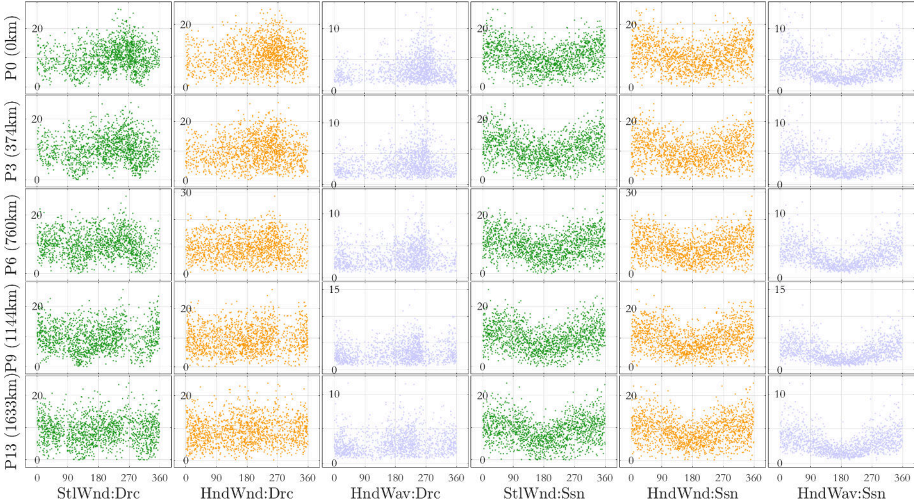
- 14 longitude-latitude pairs
- Satellite observation nearest to each pair used for each swath
- Corresponding hindcast data for each pair at time of swath
- "Instantaneous" satellite wind vector, hindcast wind vector, hindcast H_S and wave direction for 1532 times
- Most southerly location for conditioning in MSCE
- Note colour scheme

Scatter plots on physical scale



Scatter plots of registered data : StlWnd (green), HndWnd (orange), HndWav(blue)

Covariate dependence



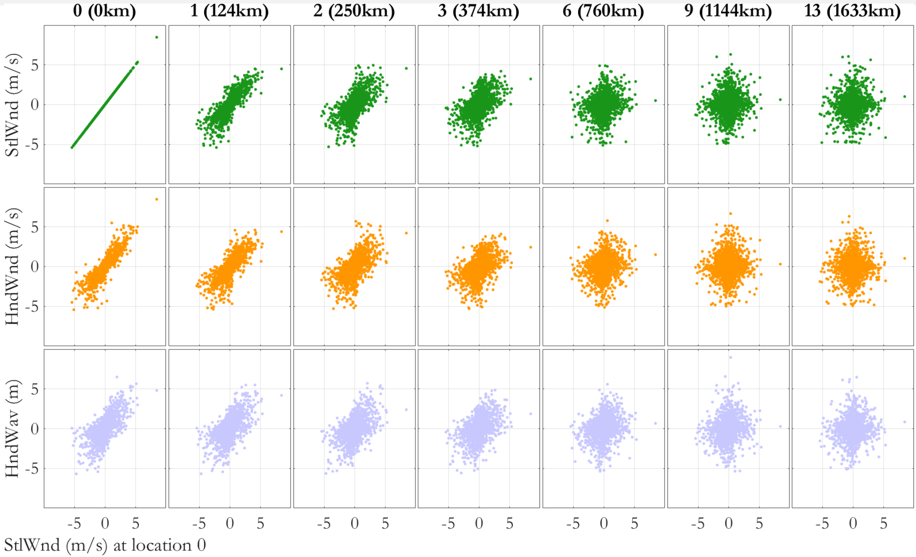
Directional and seasonal dependence. “Direction” is that from which fluid flows measured clockwise from North
StlWnd (green), HndWnd (orange), HndWav(blue)

Marginal transformation to standard Laplace scale

Procedure

- Non-stationary piecewise constant directional-seasonal marginal extreme value model (`github.com/ECSADES/ecsades-matlab`)
- Pre-specified 8 directional bins ("octants") of equal width centred on cardinal and semi-cardinal directions
- Pre-specified "summer" and "winter" seasonal bins
- Generalised Pareto model for peaks over threshold
- Model parameters vary smoothly between bins, optimal roughness found using cross-validation
- Multiple extreme value thresholds with non-exceedance probabilities between 0.7 and 0.9 considered
- Bootstrapping for uncertainties
- Uncertainty in marginal model not propagated
- Independent marginal models for pair of variable (`StlWnd`, `HndWnd`, `HndWav`) and location (0,1,...,13)

Scatter plots on Laplace scale



Registered data on Laplace scale: StlWnd (green), HndWnd (orange), HndWav(blue)

Conditional extremes

$$Y|\{X = x\} = \alpha x + x^{\beta} Z$$

- Asymptotically-motivated, Heffernan and Tawn [2004]
- $X \sim \text{Lpl}$, $Y \sim \text{Lpl}$, and $x > u$
- $\alpha \in [-1, 1]$, $\beta \in (-\infty, 1]$
- Z is a residual random variable characterised empirically, or estimated assuming $Z \sim N(\mu, \sigma^2)$, so

$$E[Y|\{X = x\}] = \alpha x + \mu x^{\beta}$$
$$\text{var}[Y|\{X = x\}] = \sigma^2 x^{2\beta}$$

- Identifiability of α and μ when $\beta \approx 1$
- Model fitting means estimating α, β, μ and σ

Spatial conditional extremes

$$\boldsymbol{X}|\{X_0 = x\} = \boldsymbol{\alpha} x + x^{\boldsymbol{\beta}} \boldsymbol{Z}$$

- Shooter et al. [2019], Wadsworth and Tawn [2019]
- $\boldsymbol{X} = (X_1, X_2, ..., X_q)$, are now observed at q points in space
- All marginal $X_k \sim$ Lpl, and $x > u$
- $\alpha_j \in [-1, 1]$, $\beta_j \in (-\infty, 1]$, $j = 1, ..., q$

$$\boldsymbol{Z} \sim \mathrm{DL}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{\delta}; \boldsymbol{\Sigma})$$

- Delta-Laplace (DL) parameters μ_j, $\sigma_j > 0$, $\delta_j > 0$, $j = 1, ..., q$
- $\boldsymbol{\Sigma}$ is a (conditional) correlation matrix with powered-exponential decay with distance between the q points (with parameters $\boldsymbol{\rho}$, $\boldsymbol{\kappa}$)

- Model fitting means estimating $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ and $\boldsymbol{\rho}$, $\boldsymbol{\kappa}$
- $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ vary smoothly with distance

Multivariate spatial conditional extremes

$$\boldsymbol{X} | \{X_{01} = x\} = \boldsymbol{\alpha} x + x^{\boldsymbol{\beta}} \boldsymbol{Z}$$

- $\boldsymbol{X} = (X_{11}, X_{21}, ..., X_{q1}, X_{12}, X_{22}, ..., X_{q2}, ..., X_{1m}, X_{2m}, ..., X_{qm})$, for m quantities observed at q points in space
- All marginal $X_{k\ell} \sim$ Lpl, and $x > u$
- $\alpha_{j\ell} \in [-1, 1]$, $\beta_{j\ell} \in (-\infty, 1]$, $j = 1, ..., q$, $\ell = 1, 2, ..., m$

$$\boldsymbol{Z} \sim \mathrm{DL}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2, \boldsymbol{\delta}; \boldsymbol{\Sigma})$$

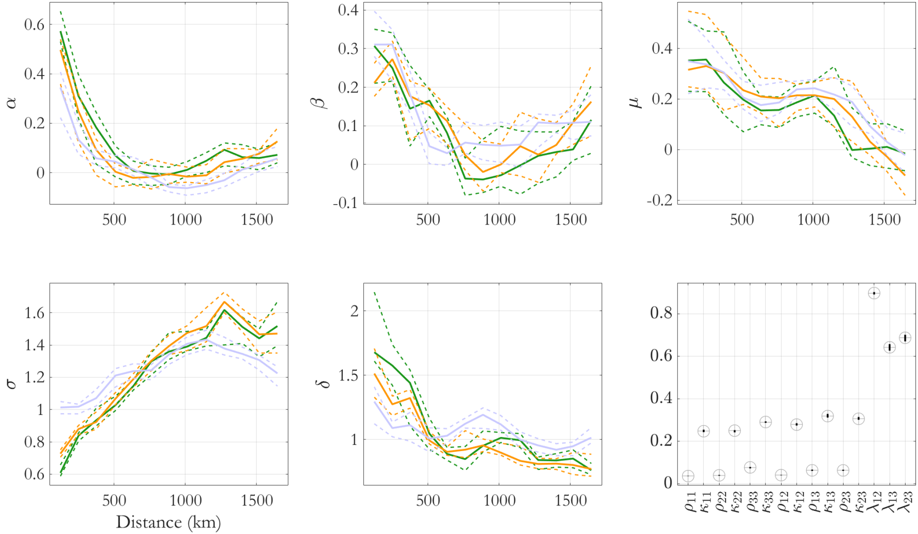
- Delta-Laplace (DL) residual parameters $\mu_{j\ell}$, $\sigma_{j\ell} > 0$, $\delta_{j\ell} > 0$
- $\boldsymbol{\Sigma}$ is a (conditional) correlation matrix with powered-exponential decay with distance between the q points for m quantities, with appropriate cross-decay (with parameters $\boldsymbol{\rho}$, $\boldsymbol{\kappa}$, $\boldsymbol{\lambda}$)

- Model fitting means estimating $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ and $\boldsymbol{\rho}$, $\boldsymbol{\kappa}$, $\boldsymbol{\lambda}$
- $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ vary smoothly with distance for each quantity

Inference

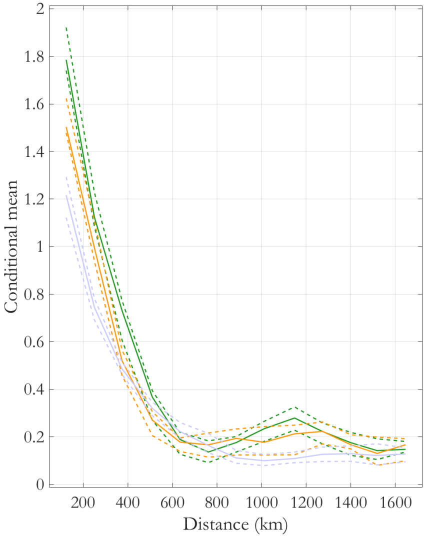
- Adaptive MCMC, Roberts and Rosenthal [2009]
- Piecewise linear forms for all parameters with distance using n_{Nod} spatial nodes
- Total of $m(5n_{\text{Nod}} + (3m+1)/2)$ parameters
- Rapid convergence, 10k iterations sufficient

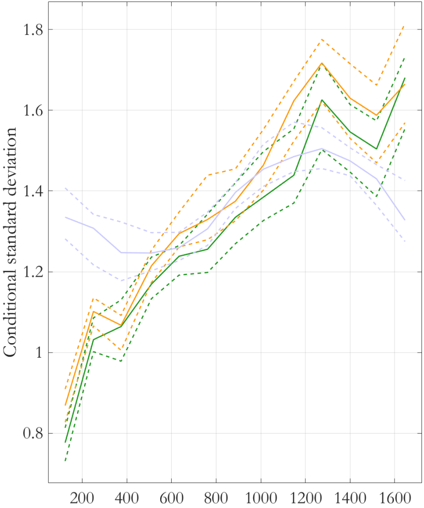
Parameter estimates for North Atlantic application



Estimates for α, β, μ, σ and δ with distance, and residual process estimates ρ, κ and λ. Model fitted with $\tau = 0.75$
StlWnd (green), HndWnd (orange), HndWav(blue)

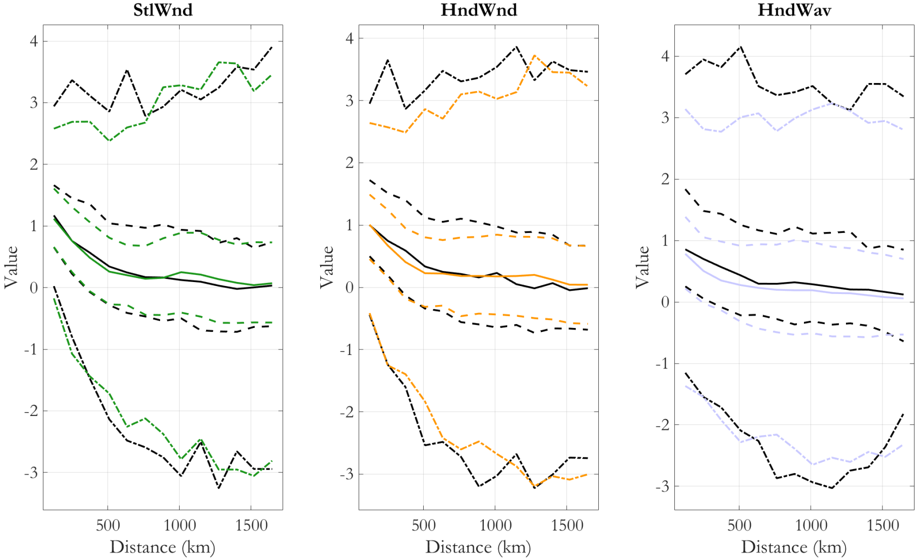
Laplace-scale conditional mean, standard deviation





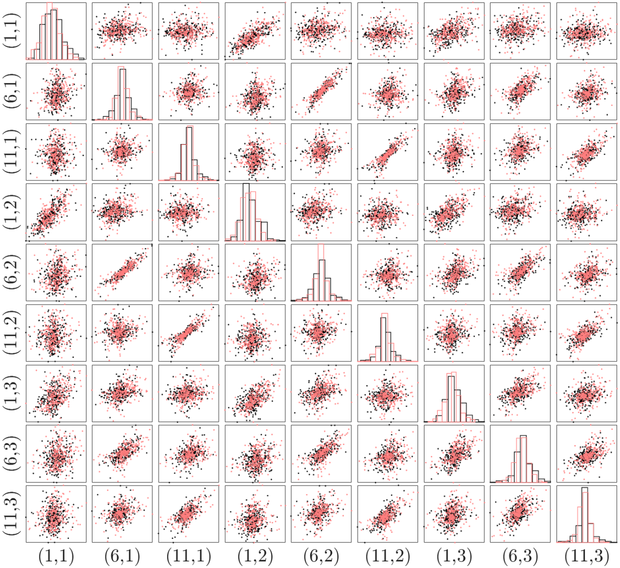
Quantile with threshold probability $\tau = 0.95$ used for illustration. Quantile level is 2.3 at zero distance on green curve
StlWnd (green), HndWnd (orange), HndWav(blue)

Laplace-scale simulation under fitted model



Threshold probability $\tau = 0.75$

Residual sample



Scatter matrix, threshold probability $\tau = 0.75$

- Black = From actual sample
- Red = Simulated from fitted model

Summary of findings

Data

- JASON and METOP jointly too sparse to use together
- 1500-2000 good instantaneous daily observations for METOP
- Sampling bias; swath time roughly the same each day
- Data are "instantaneous" not storm peak

Methodology

- Inference straightforward for $m = 3$ and $n_{\text{Nod}} = 10$
- Assumed distance-dependence structure adopted, particularly for residual, seems reasonable from diagnostics
- Marginal model (fitted independently, uncertainties not pushed through to Laplace scale); should do this jointly

Results

- Results for threshold quantile $\tau = 0.75$, other values examined
- Conditioning on other locations and quantities examined
- Spatial extent of extremal dependence for all quantities is about 600-800km

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