Environmental contours : fundamentals and recommended practice

Slides and paper at *www.lancs.ac.uk/∼jonathan*

Team ECSADES

Acknowledgement

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- DNV-GL: Ole Christian Astrup, Elzbieta Bitner-Gregersen, Erik Vanem
- HR Wallingford: Nigel Bunn, Ben Gouldby, Ye Liu
- Shell: Emma Ross, David Randell, Philip Jonathan
- Univ. Oslo: Arne Huseby

Outputs

- Ocean Engineering review paper Ross et al. [2019]
- OMAE19 conference paper
- MATLAB and R software : github.com/ECSADES
- Related "spin-off" articles

Motivation

- Rational and consistent design and assessment of marine structures

- Familiarity with specifying design in terms of return values for environmental variables

- Environmental variables are dependent in general
  - Ignoring dependence results in conservative design
  - Need to characterise joint dependence in extreme environments
- Concept of return value not uniquely defined in n-D, $n > 1$
  - Contour provides a natural extension of return value
  - No unique definition of contour (only exceedance probability is fixed)

- Full time-domain response simulation is computationally expensive
- Environmental contours provide a computationally-light approximate approach to design
  - Contour characterises environment not structural response
  - With assumptions, can make inferences about structural response

Basics

Probability of failure

Random sea state $\boldsymbol{X} = (X_1, X_2, X_3, ...)$; response R in sea state
Structural strength s; probability of failure $p_{\text{Fail}}(s)$ in sea state
Stochastic "failure boundary" $(s - R|\{\boldsymbol{X} = \boldsymbol{x}\}) = G(\boldsymbol{x})$

$$
\begin{aligned}
p_{\text{Fail}}(s) &= \Pr(R > s) = \int_r I(r > s) f_R(r) dr \\
&= \int_r I(r > s) \left[\int_{\boldsymbol{x}} f_{R|\boldsymbol{X}}(r|\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \right] dr \\
&= \int_{\boldsymbol{x}} \left[\int_r I(r > s) f_{R|\boldsymbol{X}}(r|\boldsymbol{x}) dr \right] f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \\
&= \int_{\boldsymbol{x}} \left[1 - F_{R|\boldsymbol{X}}(s|\boldsymbol{x}) \right] f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \\
&= \int_{g(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \text{ , when } G(\boldsymbol{x}) = g(\boldsymbol{x}) \text{ is deterministic function}
\end{aligned}
$$

Time-domain response simulation

$$p_{\text{Fail}}(s) = \int_{\boldsymbol{x}} \left[1 - F_{R|\boldsymbol{X}}(s|\boldsymbol{x})\right] f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}$$
$$= \int_{g(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \text{ , when } G(\boldsymbol{x}) = g(\boldsymbol{x}) \text{ is deterministic function}$$

- Always need density $f_{\boldsymbol{X}}(\boldsymbol{x})$
  - Joint density, and extreme tail behaviour important
- Ideally also need $F_{R|\boldsymbol{X}}(r|\boldsymbol{x})$
  - Might not be known or could be computationally expensive
  - $\int_{\boldsymbol{x}} \left[1 - F_{R|\boldsymbol{X}}(s|\boldsymbol{x})\right] f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}$ could be computationally expensive

Motivation for contours

$$p_{\text{Fail}}(s) = \int_{\boldsymbol{x}} \left[1 - F_{R|\boldsymbol{X}}(s|\boldsymbol{x})\right] f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} = \int_{g(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x}\text{ , when } G(\boldsymbol{x}) = g(\boldsymbol{x})$$

What can we do if $F_{R|\boldsymbol{X}}(r|\boldsymbol{x})$ route not feasible?

- Define region $\boldsymbol{x} \in \mathcal{A}$ with boundary $\boldsymbol{x} \in \mathcal{C}$ such that

  $$\Pr(\boldsymbol{X} \in \mathcal{A}) = \int_{\mathcal{A}} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} = 1 - p\text{ , with } p = \frac{1}{NT}\text{ say}$$

  for return period T and N sea states per annum
- Make (and justify) assumptions about $R|\{\boldsymbol{X} = \boldsymbol{x}\}$ or $G(\boldsymbol{x})$ such that

  $$p_{Fail}(s) = 1 - \int_{\boldsymbol{x}} F_{R|\boldsymbol{X}}(s|\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} \lessapprox 1 - \int_{\mathcal{A}} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} = p$$

- e.g. assume that
  - $R|\{\boldsymbol{X} = \boldsymbol{x}\}$ is "approximately increasing with $\boldsymbol{x}$"
  - $G(\boldsymbol{x})$ is "approximately deterministic" and convex
  - IFORM (and enhancements) provides clearest sets of assumptions

Contour frontier

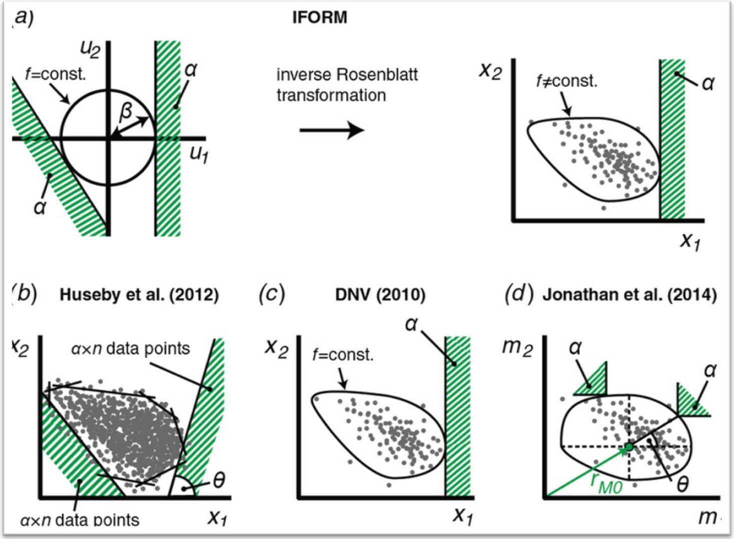
- Suppose that environmental contour boundary $\mathcal{C}$ enclosing $\mathcal{A}$ is associated with exceedance probability p
  - $\Pr(\boldsymbol{X} \in \mathcal{A}) = 1 - p$
  - $\boldsymbol{x}_{\mathcal{C}} \in \mathcal{C}$ are the "most extreme environments"
  - Assumptions imply that $R|\{\boldsymbol{X} = \boldsymbol{x}_{\mathcal{C}}\}$ for some $\boldsymbol{x}_{\mathcal{C}}$ will be the "largest response"

- If $\min\limits_{\boldsymbol{x}_{\mathcal{C}} \in \mathcal{C}}(g(\boldsymbol{x}_{\mathcal{C}})) \geq 0$ (or $\int_{\mathcal{C}} F_{R|\boldsymbol{X}}(s|\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \approx 1$), then structure does not fail on $\mathcal{C}$
  - Assumptions imply that structure therefore does not fail anywhere in $\mathcal{A}$; therefore $p_{Fail}(s) \lessapprox p$
  - If $\min\limits_{\boldsymbol{x}_{\mathcal{C}} \in \mathcal{C}}(g(\boldsymbol{x}_{\mathcal{C}})) < 0$ (or $\int_{\mathcal{C}} F_{R|\boldsymbol{X}}(s|\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} < 1$), increase p and repeat

- Alternatively use $F_{R|\boldsymbol{X}}(r|\mathcal{C}^*) = \int_{\mathcal{C}^*} F_{R|\boldsymbol{X}}(r|\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}$ as a proxy for $F_R(r)$, where $\mathcal{C}^*$ might be a restricted / modified frontier region
  - Logic for this is that $F_{R|\boldsymbol{X}}(r|\boldsymbol{x}) = 1$ for $x \in \mathcal{A} \setminus \mathcal{C}^*$ and values of r near s

Defining $\mathcal{A}$ and $\mathcal{C}$

Haselsteiner et al. [2017]



What can go wrong?

- Cannot / do not estimate $f_X(\boldsymbol{x})$ and its tails well
  - Poor modelling of marginal/conditional or marginal/dependence distributions
  - Covariates, multi-modality
  - Uncertainty propagation
- Poor (rather arbitrary) choice of $\mathcal{A}$ and $\mathcal{C}$
  - Constant density (different scale $\Rightarrow$ different density)
  - Constant non-exceedance (tangent plane, quadrant, other)
- $R|\{\boldsymbol{X} = \boldsymbol{x}\}$ is not "approximately increasing with $\boldsymbol{x}$" or $g(\boldsymbol{x})$ not convex
  - Large R for intermediate $\boldsymbol{X}$, e.g. resonance
- $R|\{\boldsymbol{X} = \boldsymbol{x}\}$ and $G(\boldsymbol{x})$ not "approximately deterministic"
  - Wrong choice of $\boldsymbol{X}$ (some important variables missing?)
  - Complex stochastic response
  - Short-term variability

Recommendations

When to use environmental contours

Nature of responses and environmental variables are known

- The dominant structural responses are all known
- The dominant environmental variables driving each structural response are all known
- The value of response is dominated by long-term variability of the environmental variables: extreme environments produce extreme responses
- The influence of short-term environmental variability is relatively small

When to use environmental contours

Response-based analysis is not possible

- There are no adequate computationally-efficient structural response models available
- There are computationally-demanding structural response models available, but no time or expertise to develop approximate structural response models, including generic load models

At outline design stage

- The specifics of the structure are not known

How to use environmental contours wisely

Reality check

- Environmental contours are approximate method for design that can only provide approximations to extreme responses
- The use of contour approaches may need to be supported in final design by full long-term analysis

Sufficient environmental data available

- There are sufficient historical data available to estimate the joint distribution of all these environmental variables adequately

How to use environmental contours wisely

Estimate more than one environmental model, and consider the sensitivity of the model to arbitrary modelling choices

- The sensitivity of environmental contour estimates to arbitrary choices made when estimating a model for the joint distribution of environmental parameters should be investigated
- When different equally-plausible environmental models provide different contour estimates, the current research suggests that all contours should be considered valid and used together for choice of environmental values corresponding to extreme responses
- Be concerned when two different environmental models provide materially different contour estimates using a common contouring approach

How to use environmental contours wisely

If unsure which contour to use, estimate more than one type

- Each type of environmental contour is seeking to achieve different objectives
- Consider estimating contours of different types, and establish approximate consistency of inferences from different contours
- Some contour methods (e.g. ISORM Chai and Leira 2018, highest density contours Haselsteiner et al. 2017) are more likely to provide conservative estimates, since they seek to exploit convex characterisations of the failure surface

How to use environmental contours wisely

Choose multiple points from the environmental contours for response evaluation

- Multiple combinations of values of environmental variables falling on or near the frontier interval of the environmental contour should be used
- When the frontier interval is not known, a wide set of combinations of values of environmental variables on or near the environmental contour should be used
- If in doubt, choose more points and choose points more widely

How to use environmental contours wisely

Consider other sources of uncertainty

- How influential are the effects of covariates (directionality, seasonality)?
- Have all environmental variables influencing the response been considered in the environmental model and contours?
- What would be the effect of propagating uncertainty from model for environment to contours to design specification?

Final thoughts

Final thoughts

- Should we be quoting return values at all?
- Emulation and uncertainty propagation
- Decision theory

References

W. Chai and B. J. Leira. Environmental contours based on inverse sorm. *Marine Structures*, 60, 2018.

A. F. Haselsteiner, J.-H. Ohlendorf, W. Wosniok, and K.-D. Thoben. Deriving environmental contours from highest density regions. *Coastal Eng.*, 123:42–51, 2017.

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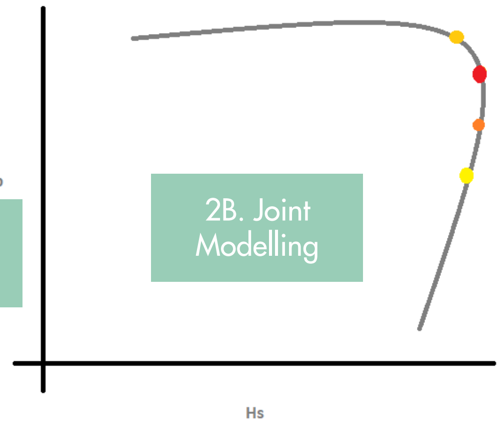
Applying Contours

Step 1	Choice of environmental data
Step 2	Marginal and joint modelling of environment
Step 3	Deriving environmental contour
Step 4	Running time-dmn sim. for selection of points on contour
Step 5	Adjustment to account for STV, model mis-spec

1. Select Data

Environment

3. Derive the contour



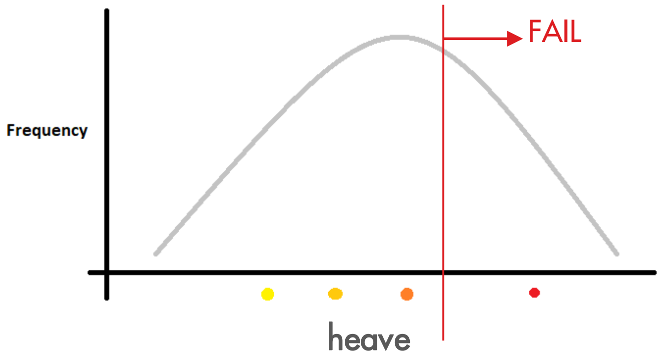
2A. Marginal modelling

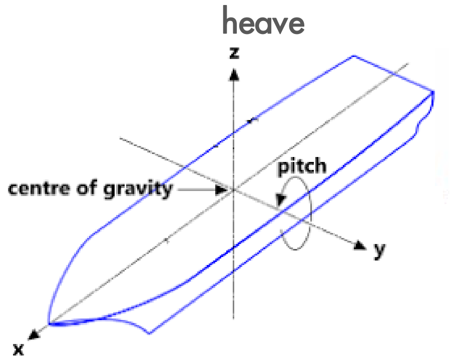
2B. Joint Modelling

2A. Marginal modelling

4. Run time-domain simulation for subset of points along contour

Response

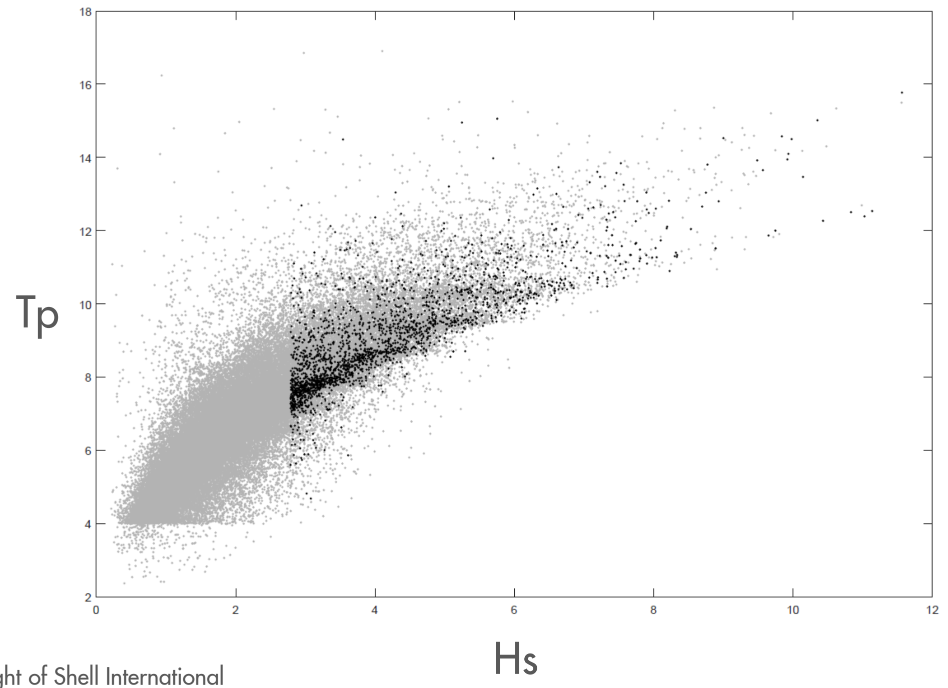


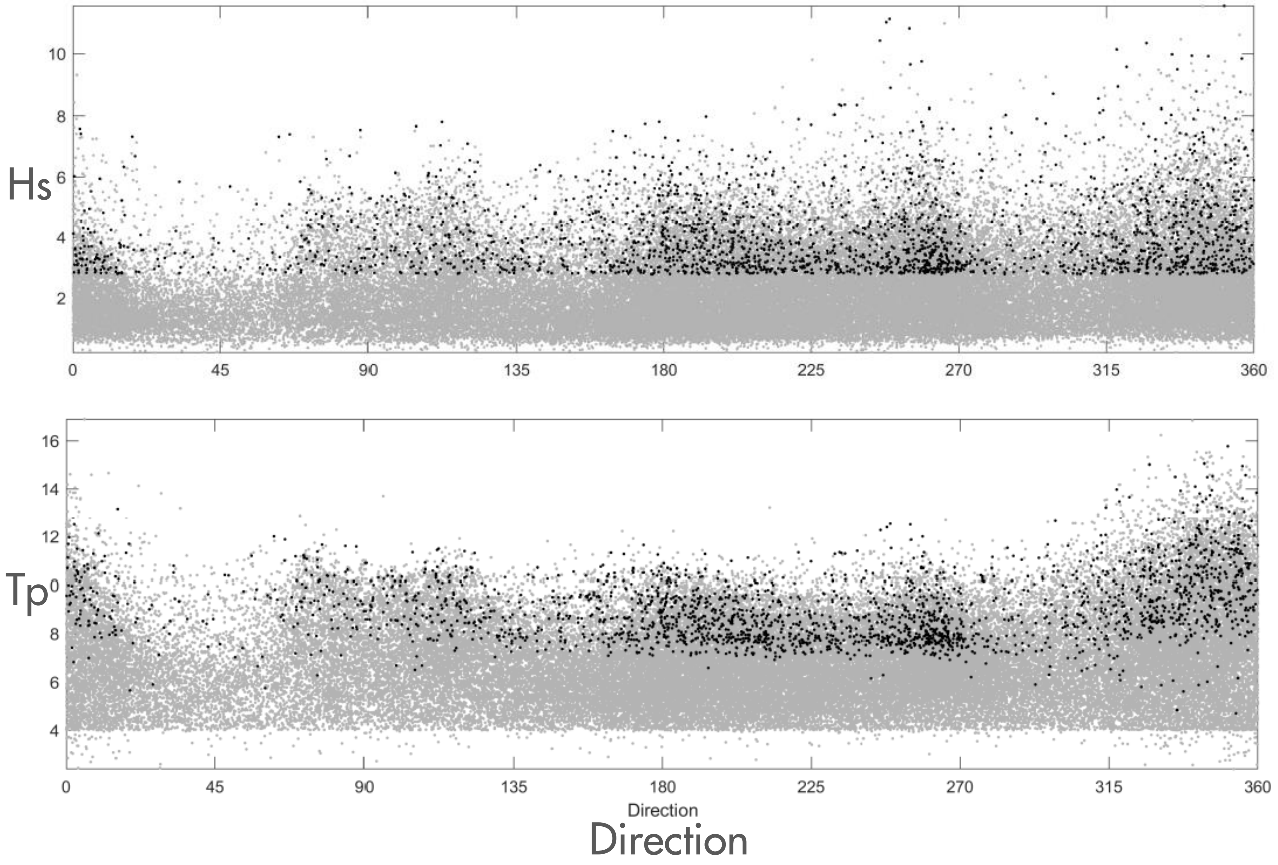


5. Make Adjustments: for STV and/or model mis-spec

Case Study: Data for the Environment

- Large historical sample of sea-state Hs,Tp for northern North Sea 1979-2013, NORA10-WAM hindcast = ~100,000 3-hourly wave data at particular location
- Peak pick Hs, taking associated peak-Tp

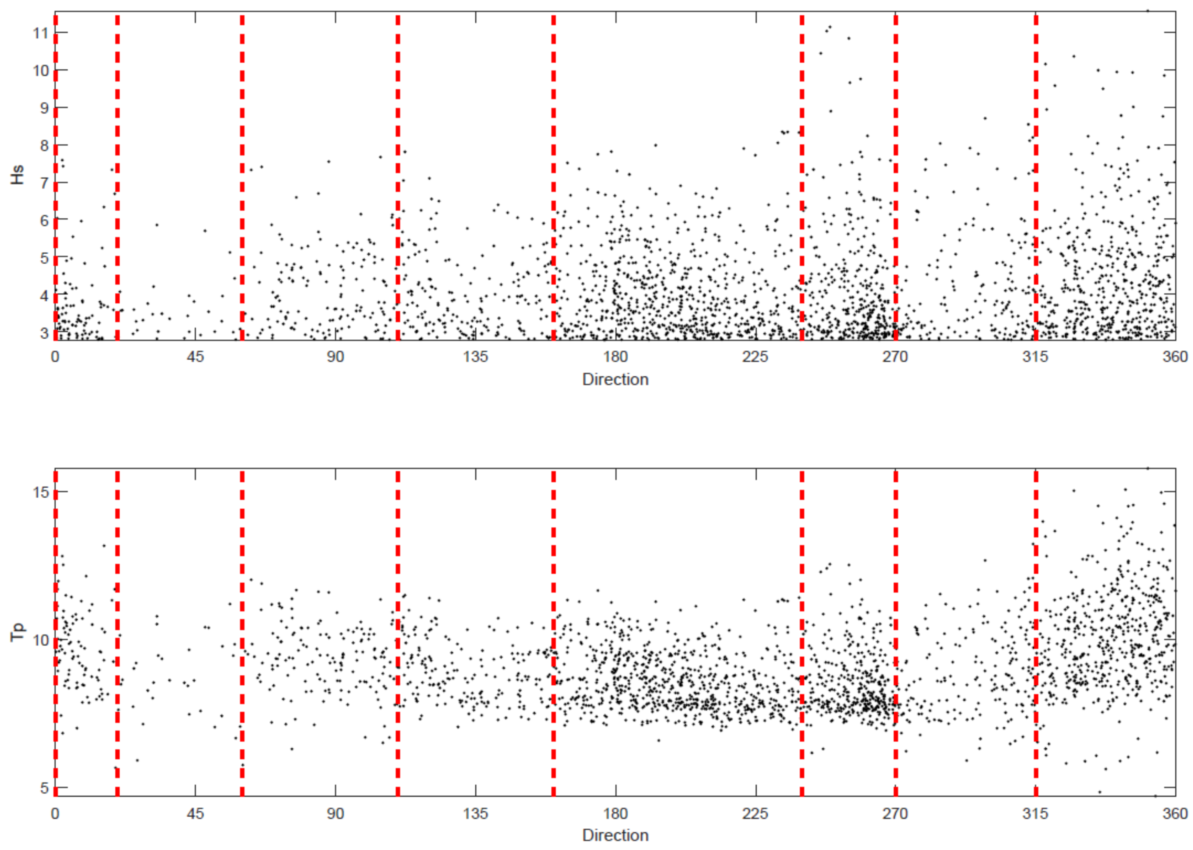




Step 2 **Marginal** and joint modelling of environment

Marginal Model

- Accounting for covariate effects in the model
- Penalised Piecewise Constant (PPC) model



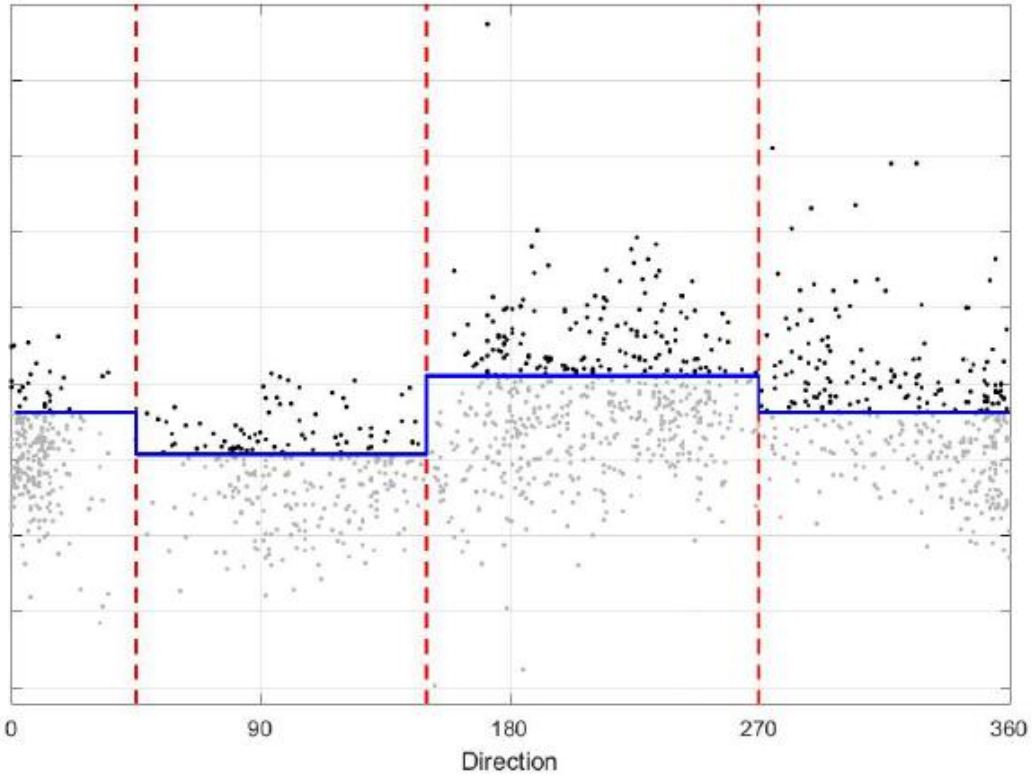
Marginal Model: PPC Explained

- Likelihood

$$\mathcal{L} = \prod_{b=1}^{B} \prod_{\substack{i;A(i)=b;\\ \dot{y}_i>\psi_b}} f_{GP}(\dot{y}_i \mid \xi, \nu_b, \psi_b)$$

- Log likelihood

$$\ell(\lambda) = \log \mathcal{L} + \lambda \left(\frac{1}{B}\sum_{b=1}^{B} \nu_b^2 - \left[\frac{1}{B}\sum_{b=1}^{B} \nu_b \right]^2 \right)$$



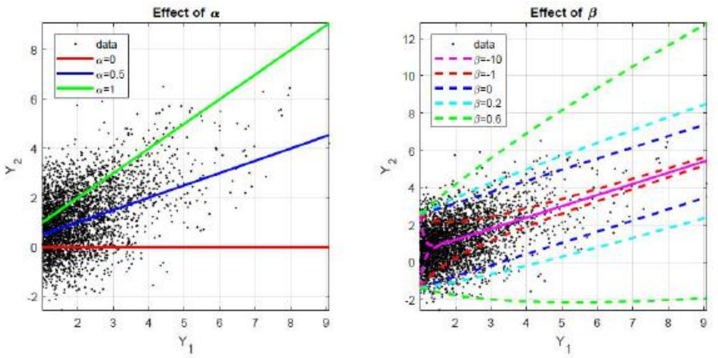
- Maintain 2 sources of uncertainty throughout: sample uncertainty (bootstrapping); and threshold choice (try an ensemble of NEPs)

Joint Model: Heffernan & Tawn

- Conditional extremes: Heffernan & Tawn
  1. Use the fitted marginal models to transform to standard (Laplace or Gumbel) scale, using Probability Integral Transform
  2. Fit regression-like relationship motivated by asymptotic theory
  3. Simulate under the model, use to estimate return-values

$$(Y_2|Y_1 = y) = \alpha y + y^{\beta} W$$

- $Y_2 =$ Tp $\quad Y_1 =$ Hs *on Gumbel scale*
- for $y >$ sufficiently large threshold ϕ
- $\alpha \in [0,1]$, $\beta \in (-\infty, 1]$
- $W \sim N(\mu, \sigma)$



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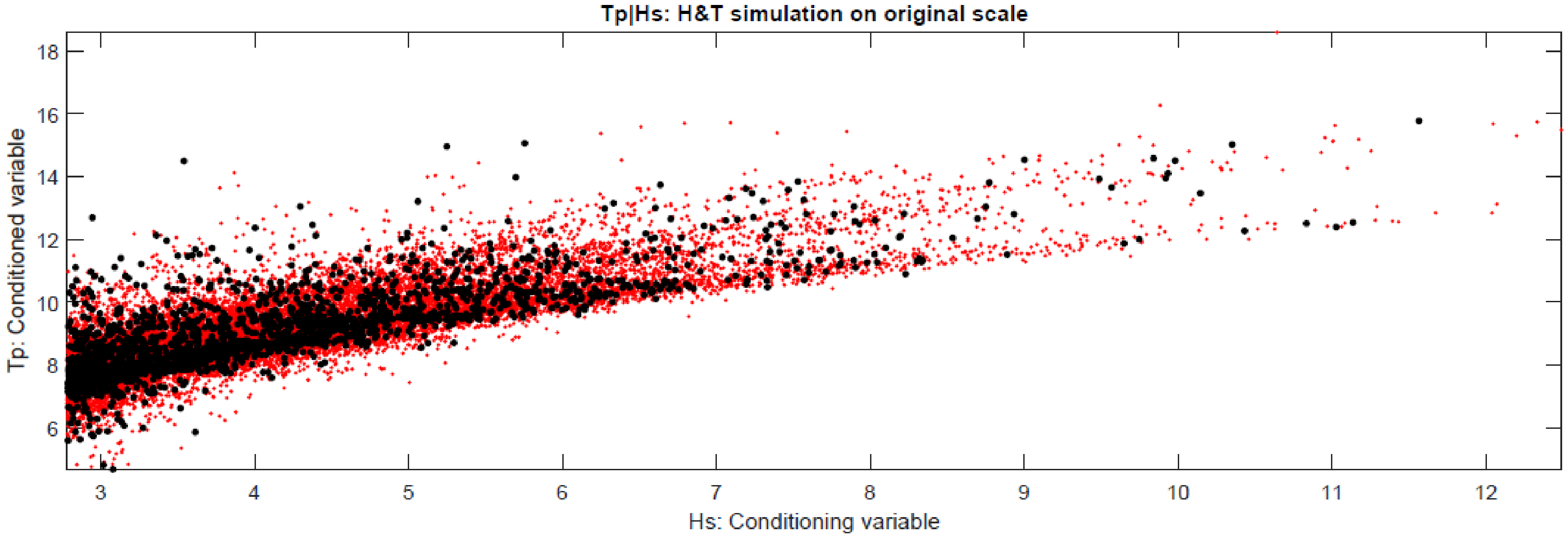
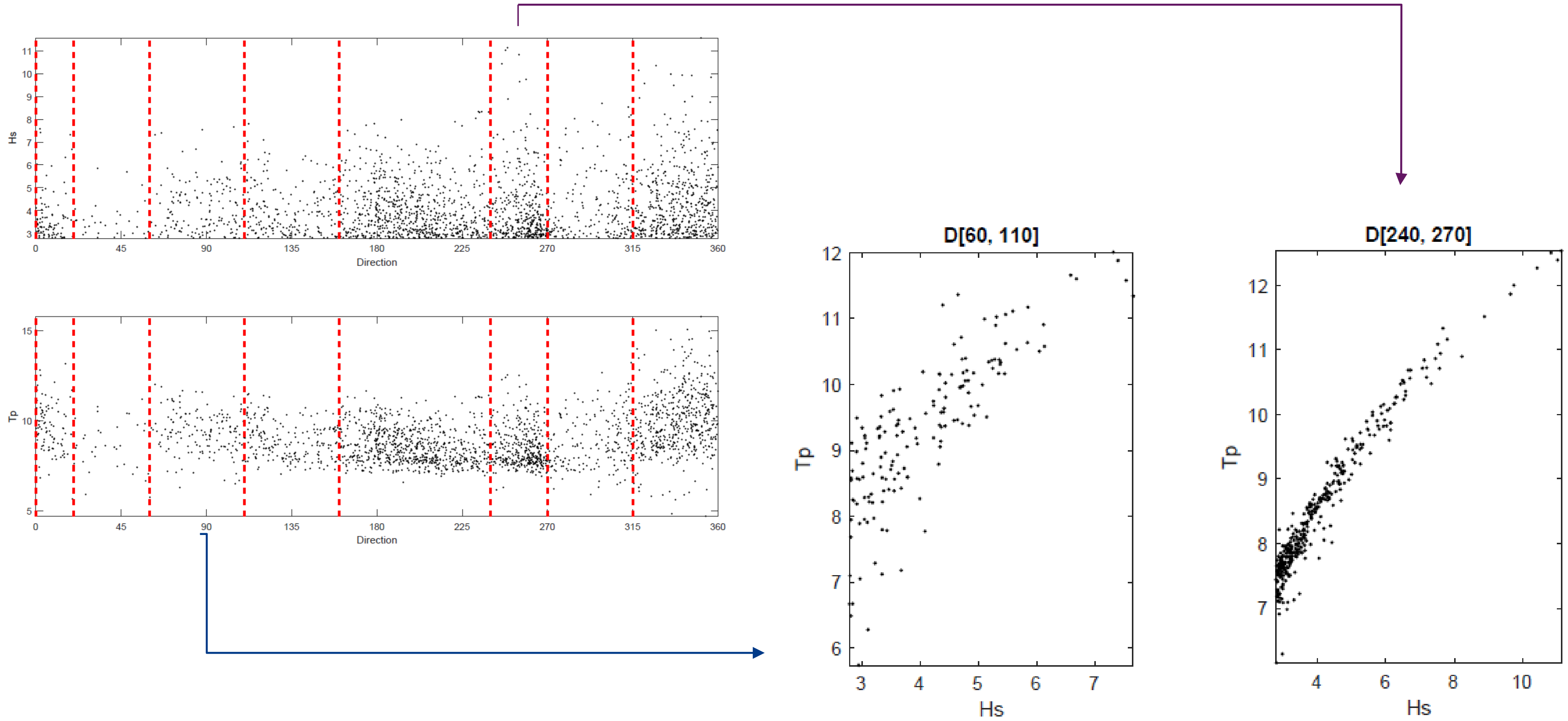


Directional Covariate in Joint Model



- Similar to marginal model: use penalized piecewise constant Heffernan & Tawn parameters $(Y_2|Y_1 = y) = \alpha y + y^{\beta} W$

Contours

- Direct Sampling (Huseby et al. 2012)
- Constant exceedance (orange)
- Isodensity (blue)
- A contour each for [20,30,40,50,70,100,200] year

- All defined from starting lock point = median of N-year maximum distribution for Hs, and conditional median in Tp